LONGEST COMMON EXTENSIONS IN SUBLINEAR SPACE

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CPM 2015
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THE LONGEST COMMON EXTENSION PROBLEM

Prepreprocess T of length n to support the query:

LCE(i,j): return the length of the longest common prefix of T[i…n] and T[j…n]

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Example

	1	2	3	4	5	6	7	8	9	10	11
T =	A	C	A	C	B	A	C	B	A	C	C

LCE(3,6)=5

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Suffix 3			A	C	B	A	C	B	A	C	C
Suffix 6						A	C	B	A	C	C

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TWO SIMPLE SOLUTIONS

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ℓ=LCE(i,j)

		Space	Time
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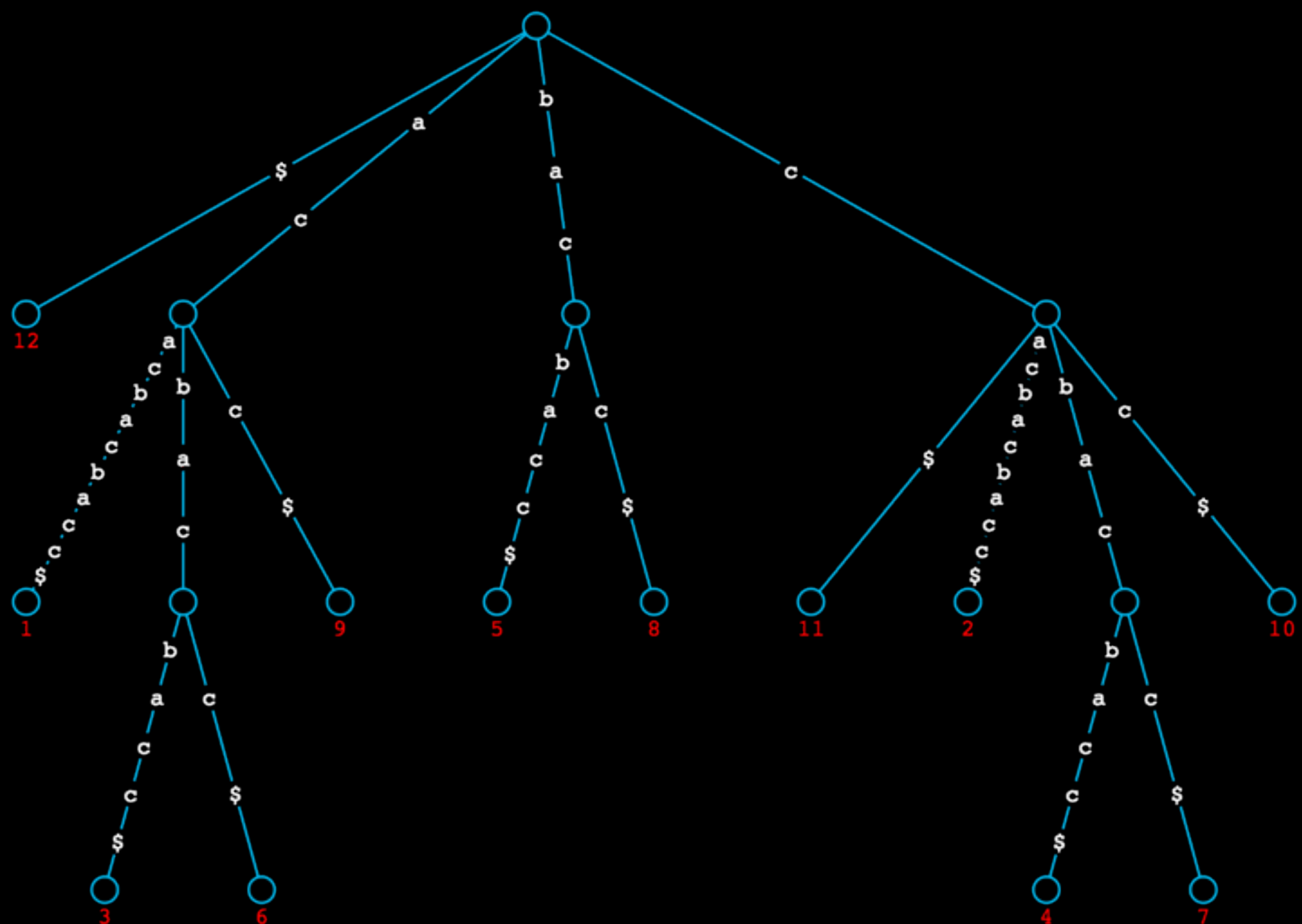

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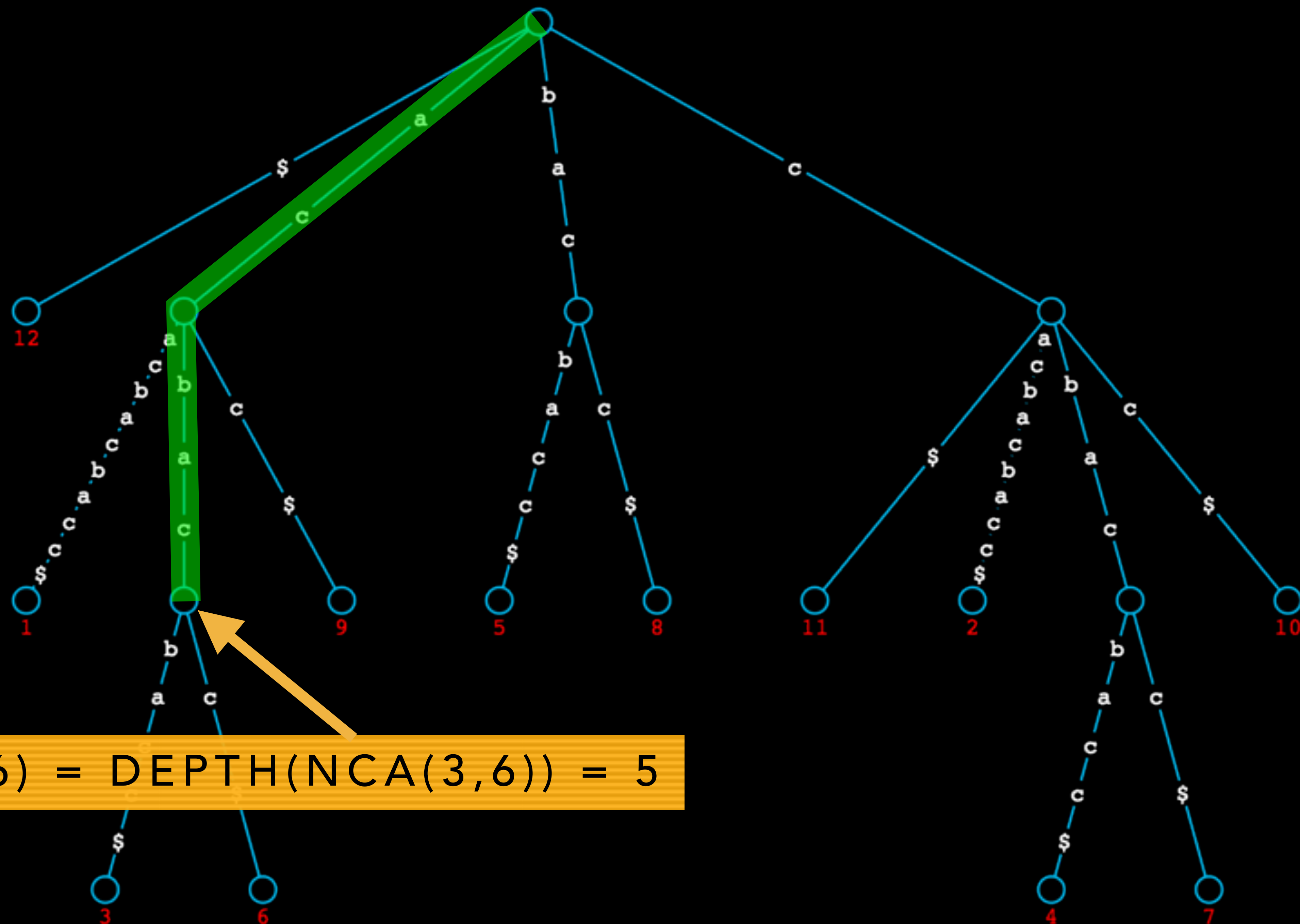

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OUR RESULTS

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Can we obtain $O(n/\tau)$ space and $O(\tau)$ time for all $1 \leq \tau \leq n$?

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CPM 2012 RESULTS*

		Space	Time	Trade-off range
3	Deterministic trade-off	$O(n/\tau)$	$O(\tau^2)$	$1\leq\tau\leq\sqrt{n}$
4	Randomized trade-off	$O(n/\tau)$	$O(\tau\log(\ell/\tau))$	$1\leq\tau\leq n$

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*Philip Bille, Inge Li Gørtz, Benjamin Sach, Hjalte Wedel Vildhøj, Time-Space Trade-Offs for Longest Common Extensions, CPM 2012

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CPM 2015 RESULTS

		Space	Time	Trade-off range
5	NEW deterministic trade-off	$O(n/\tau)$	$O(\tau \log^2(n/\tau))$	$1/\log n \leq \tau \leq n$
6	NEW randomized trade-off	$O(n/\tau)$	$O(\tau)$	$1 \leq \tau \leq n$

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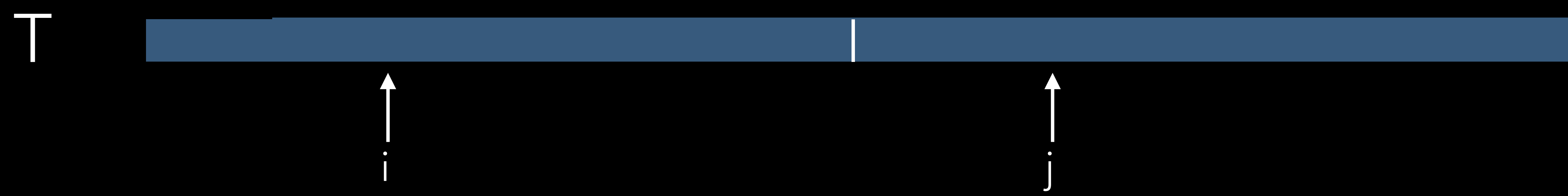
THE NEW DETERMINISTIC TRADE-0FF

TWO STRUCTURES

Data Structure 1: $O(n/\tau)$ space and $O(\tau)$ time, but works only if $|i-j|<\tau$

Data Structure 2: $O(n/\tau)$ space and $O(\tau\log^2(n/\tau))$ time:
Reduces an LCE(i,j) query to another query LCE(i',j') s.t. $|i'-j'|<\tau$

DETERMINISTIC TRADE-OFF $|i-j| \geq \tau$



Lemma
An LCE(i,j) query where i and j are in separate halves of T can be reduced to another LCE(i’,j’) query such that i’ and j’ are in the same half of T

Proof

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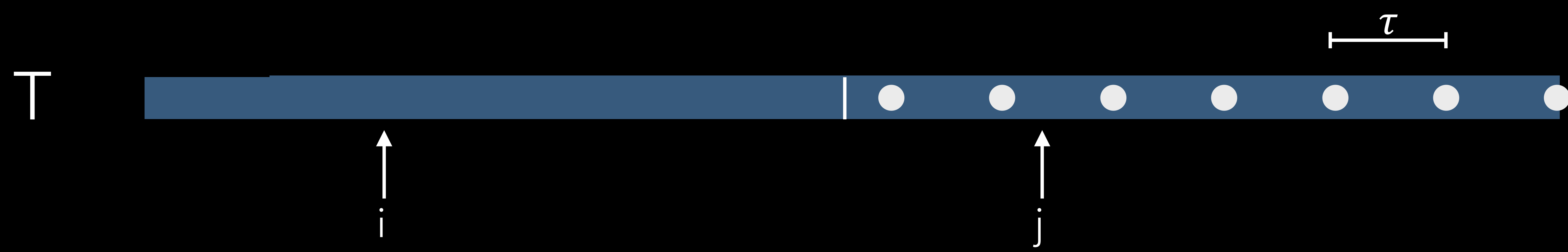

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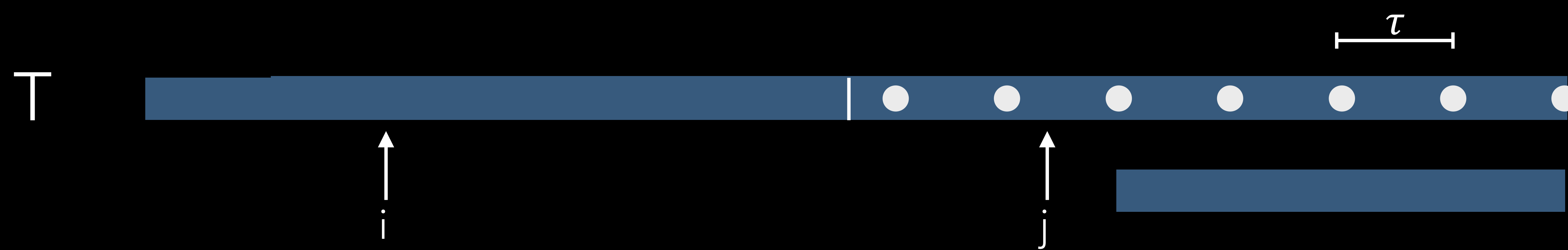

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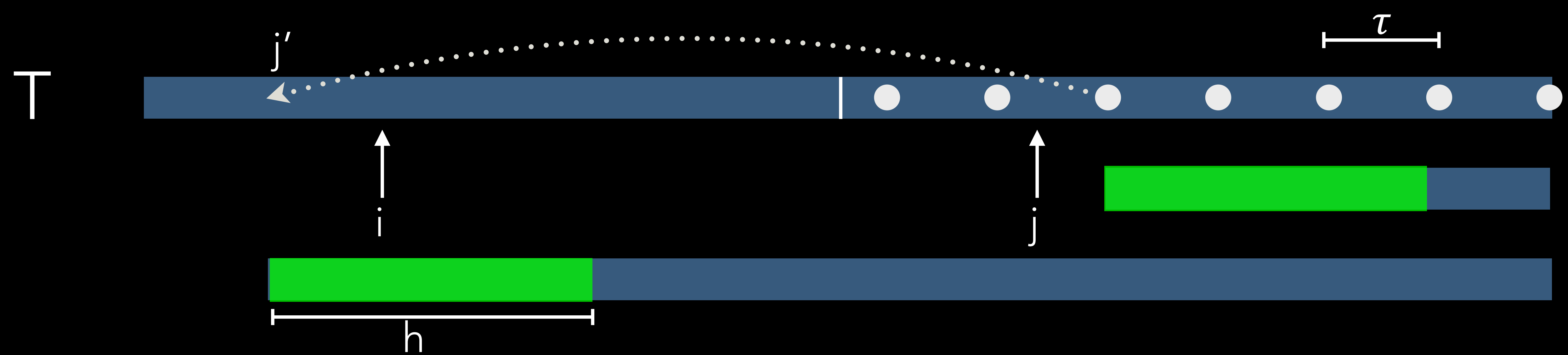

The suffix in the left half that maximizes the LCE value

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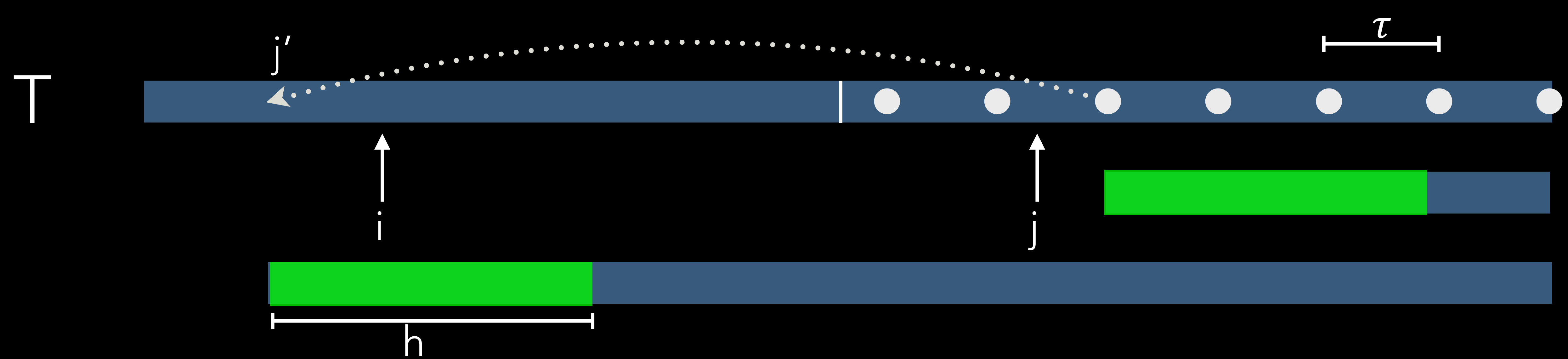

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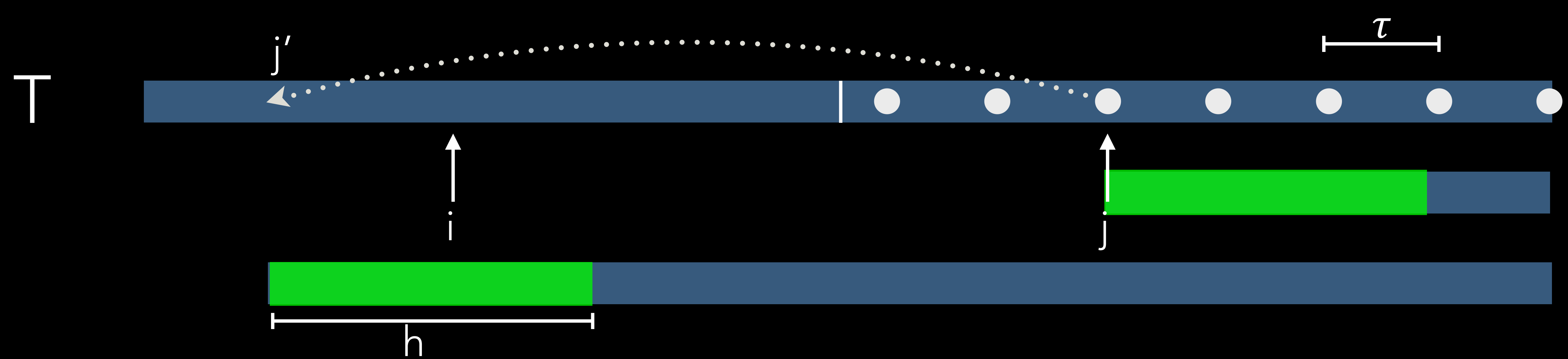

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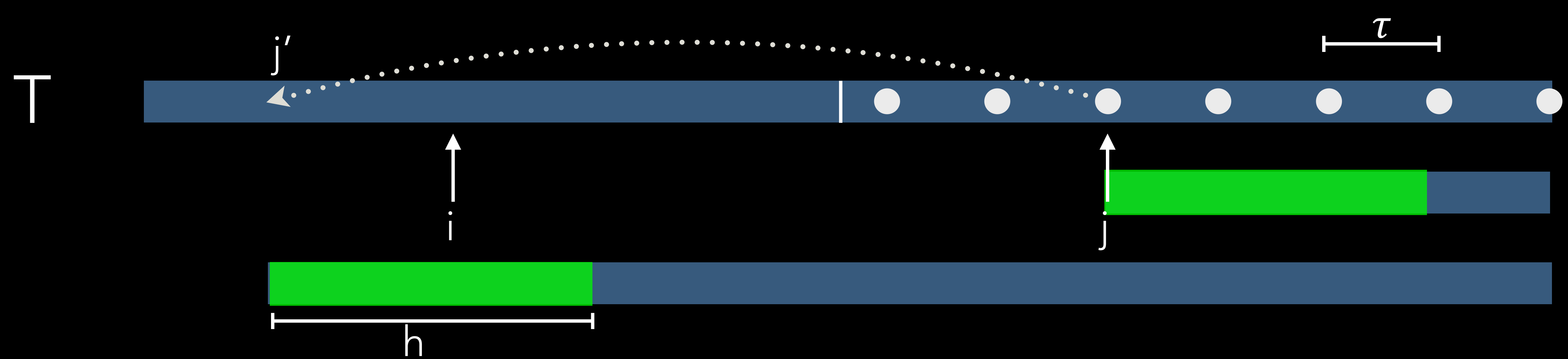

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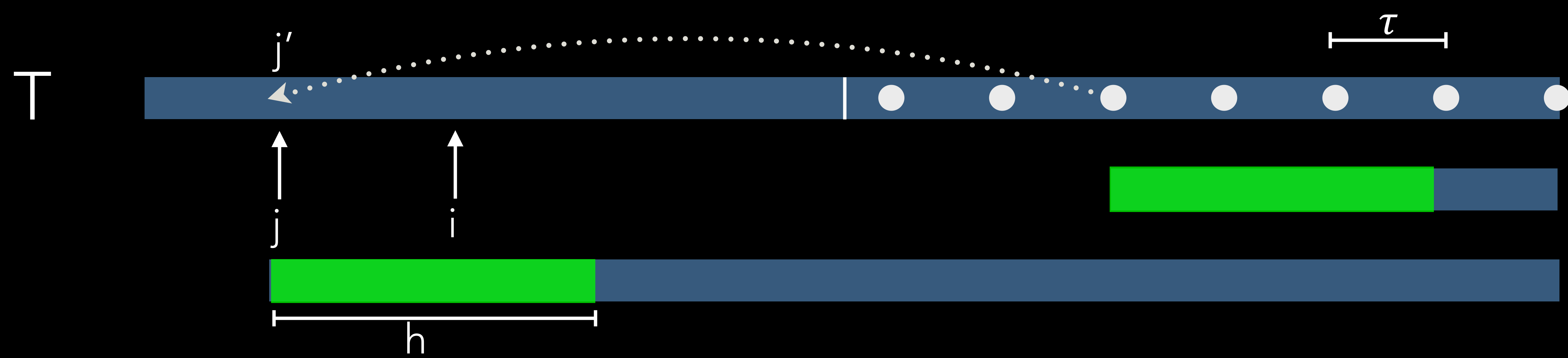

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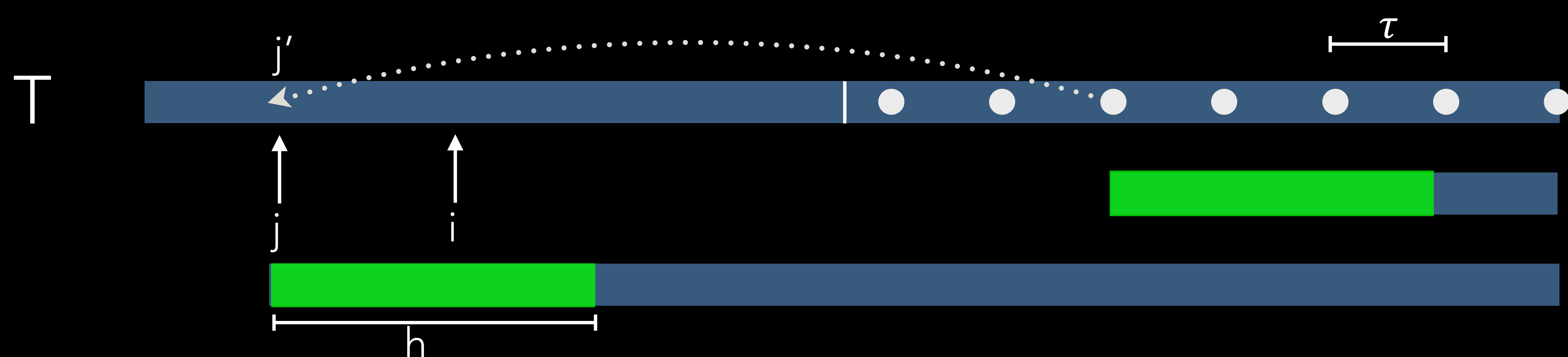

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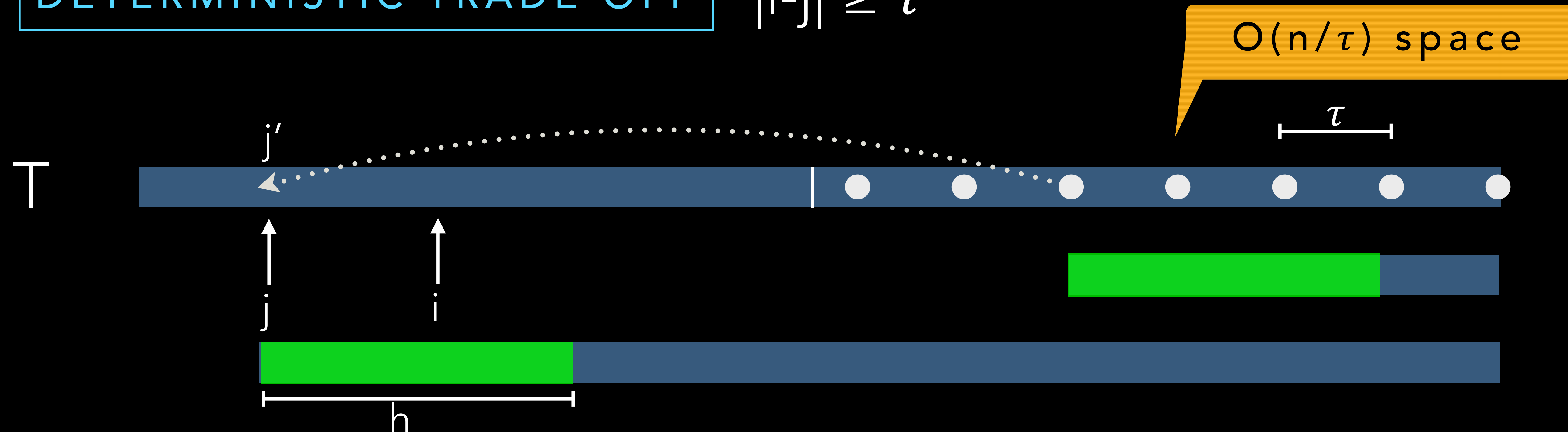

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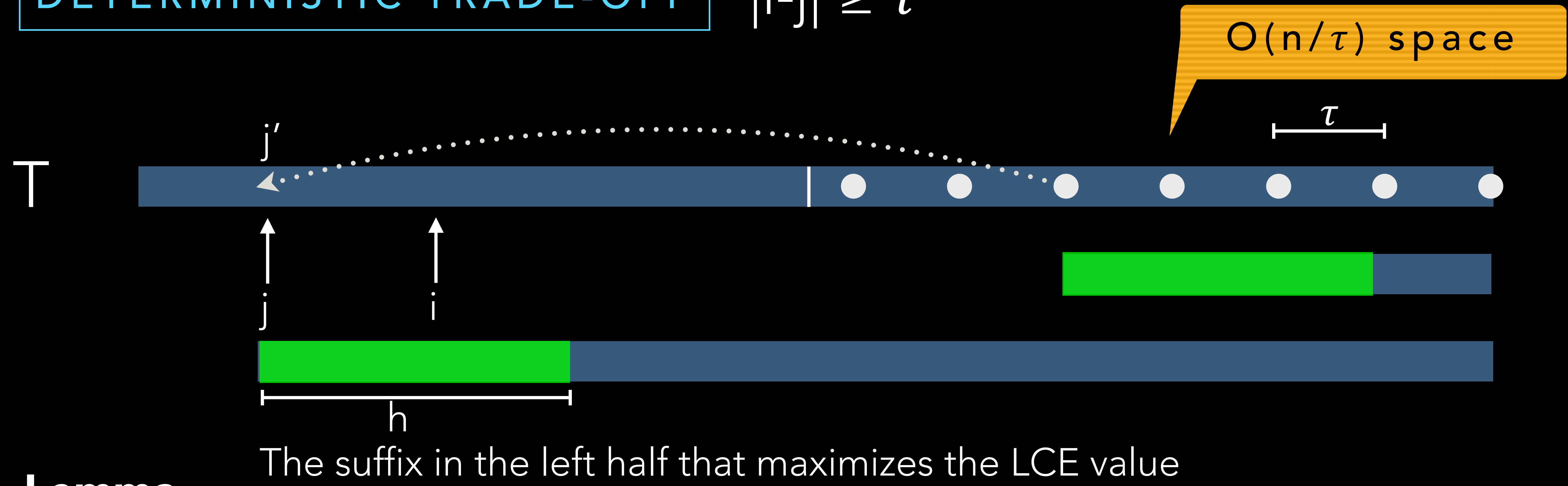

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O(τ) time

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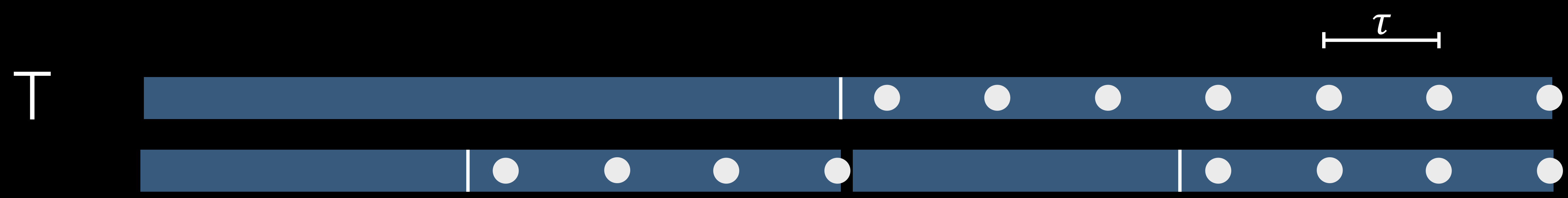

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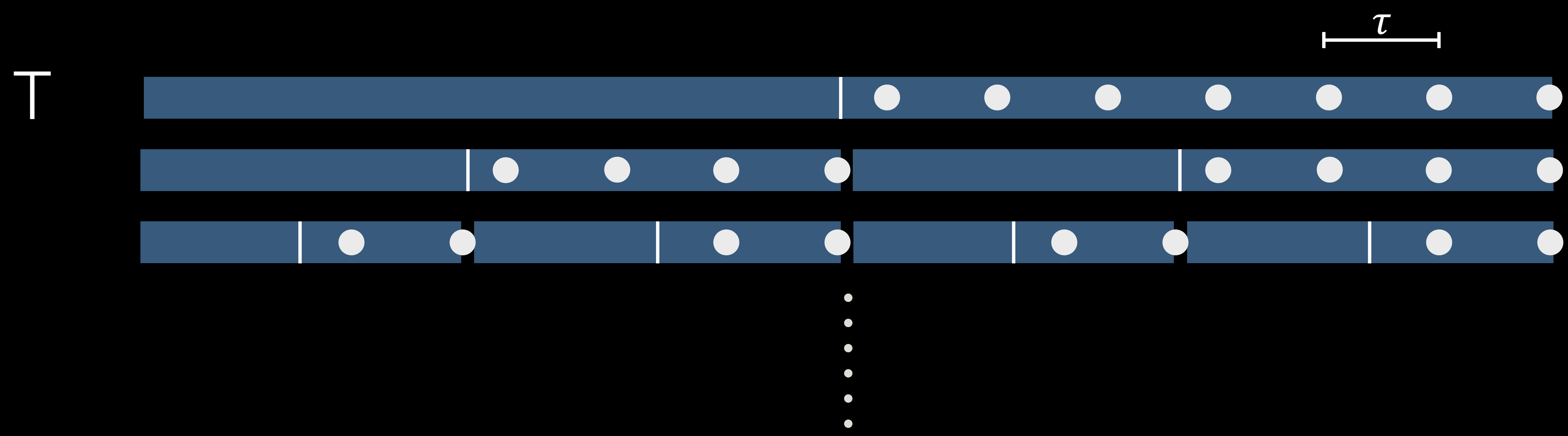

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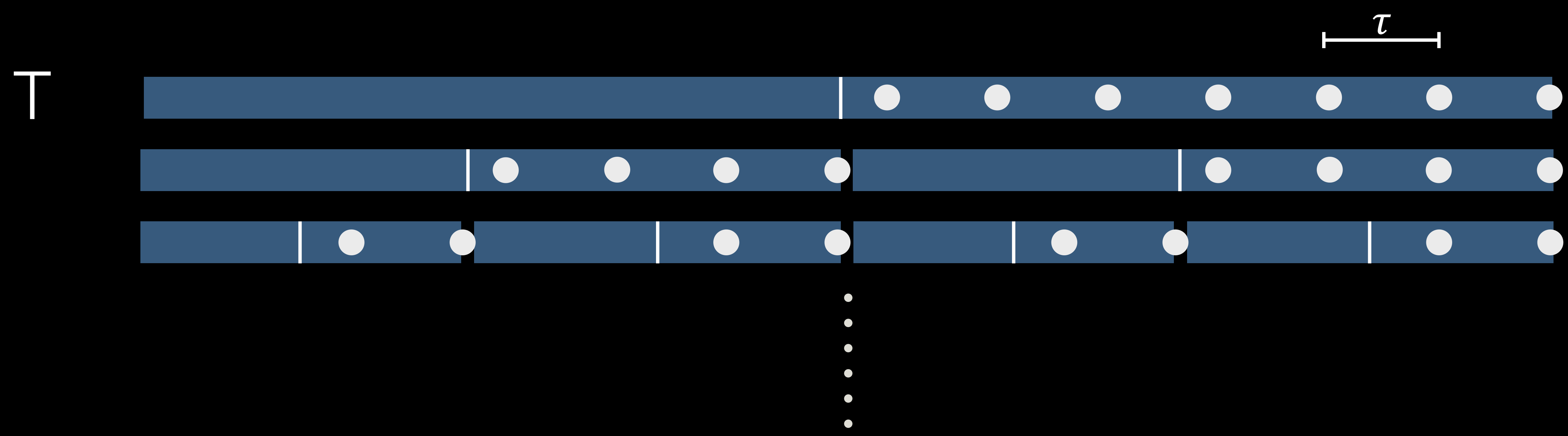

- Build data structure recursively for left and right half of T
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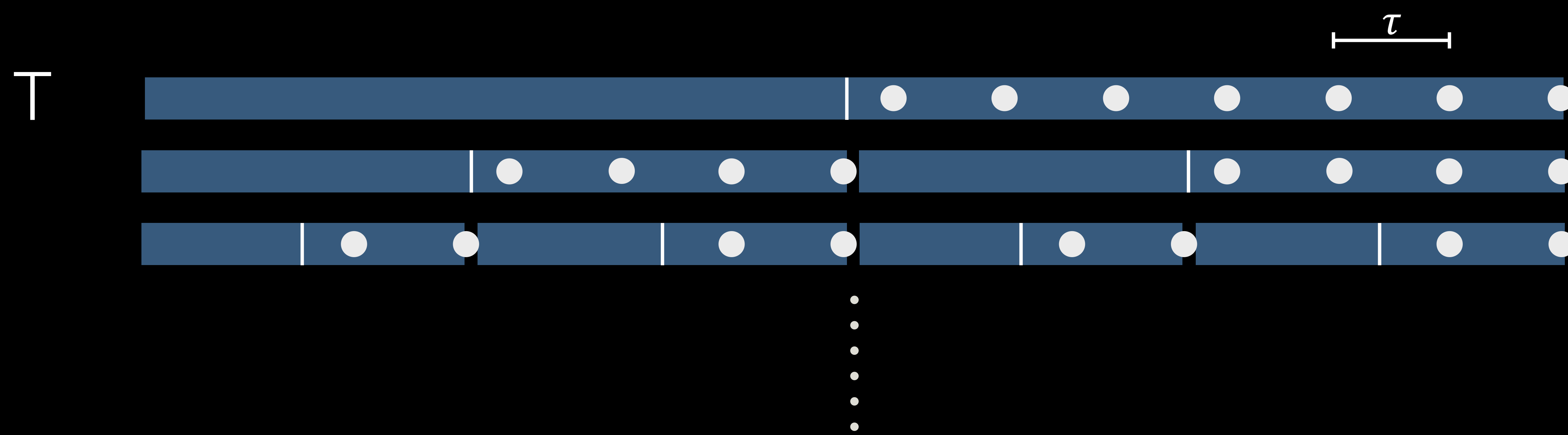

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Analysis

- $n/(2\tau)$ sampled positions on each level
- $\log(n/\tau)$ levels
- $O(\tau)$ time on each level

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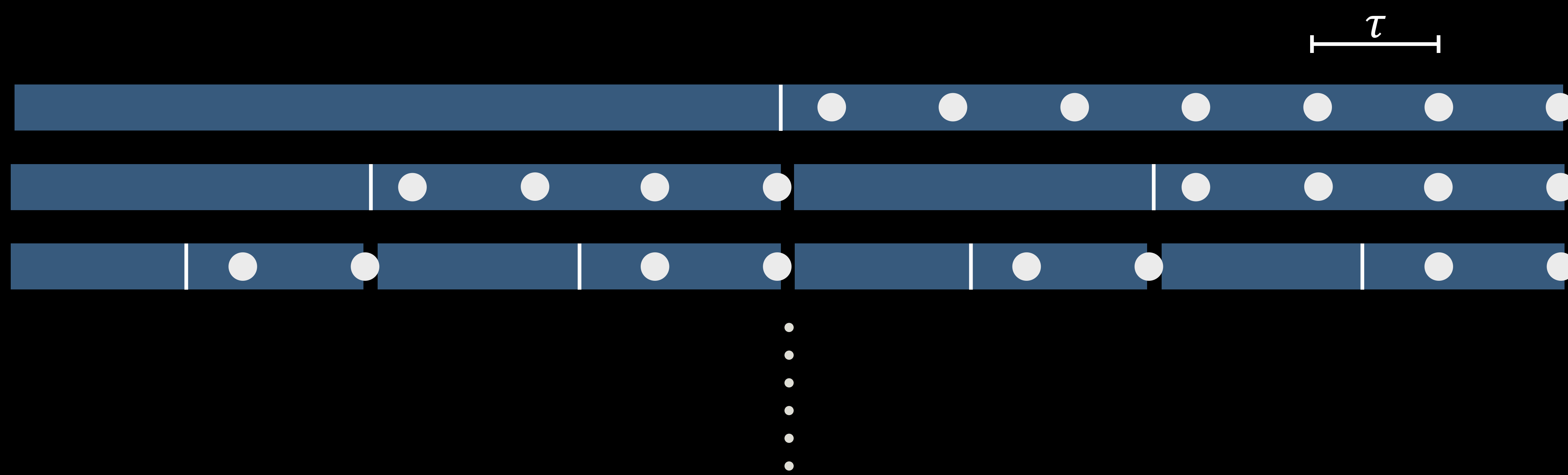

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$O((n/\tau)\log(n/\tau))$ space
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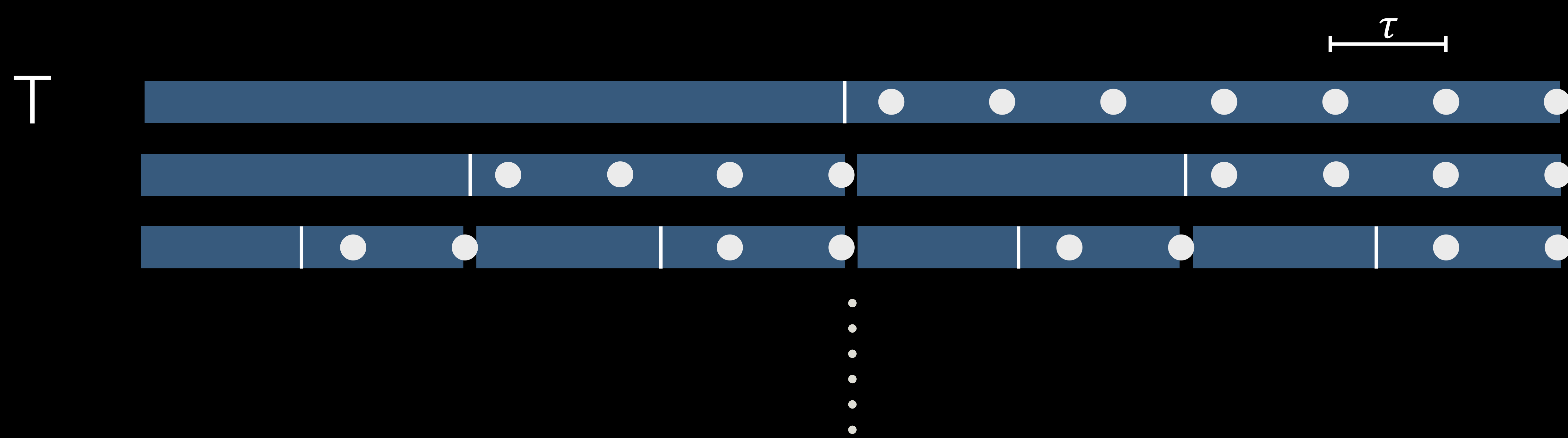

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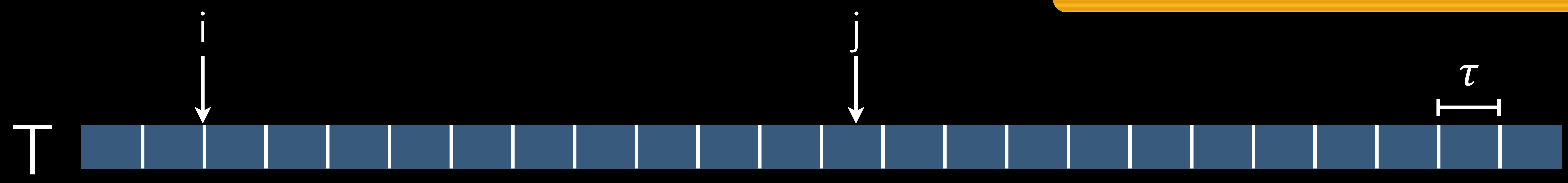

Answering an LCE query

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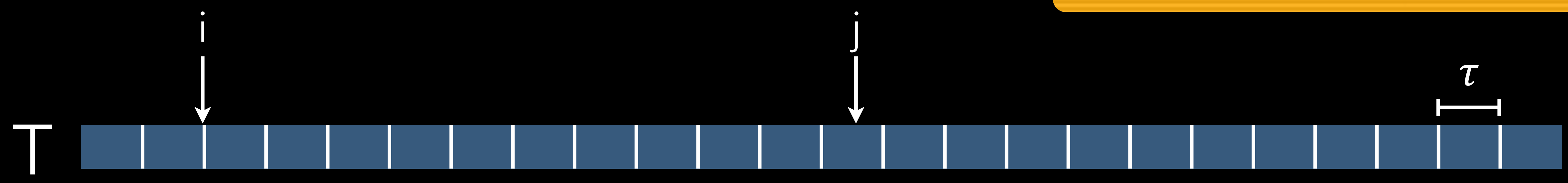

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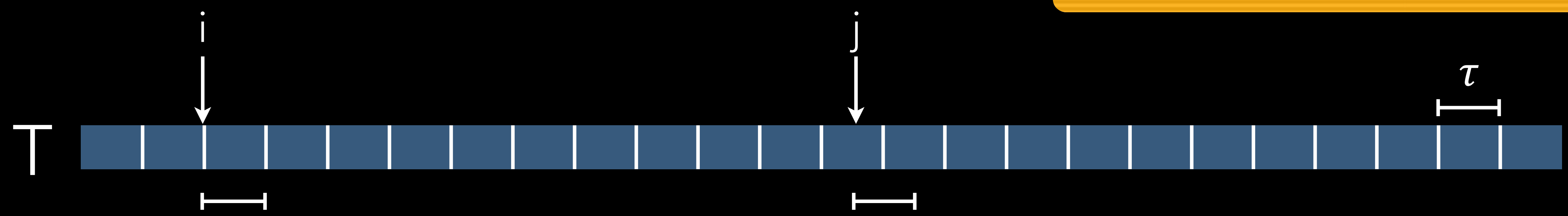

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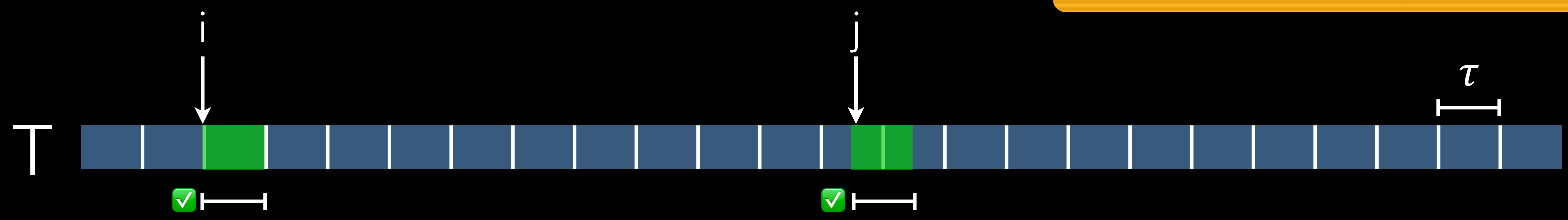

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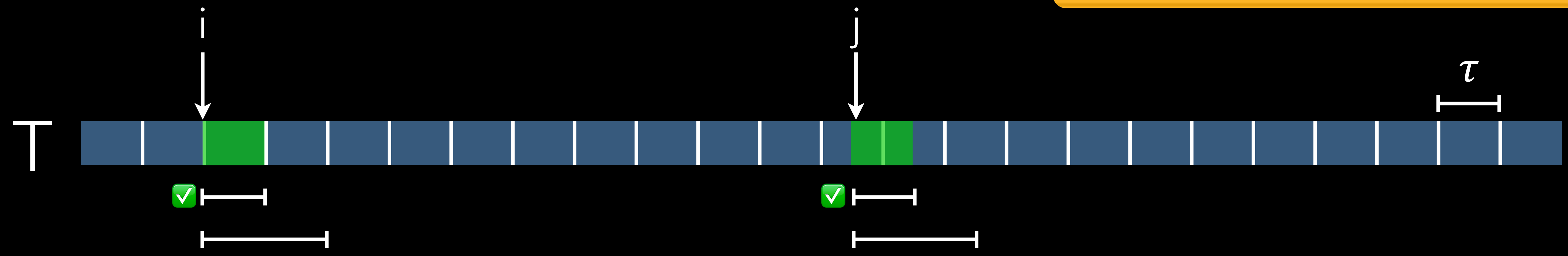

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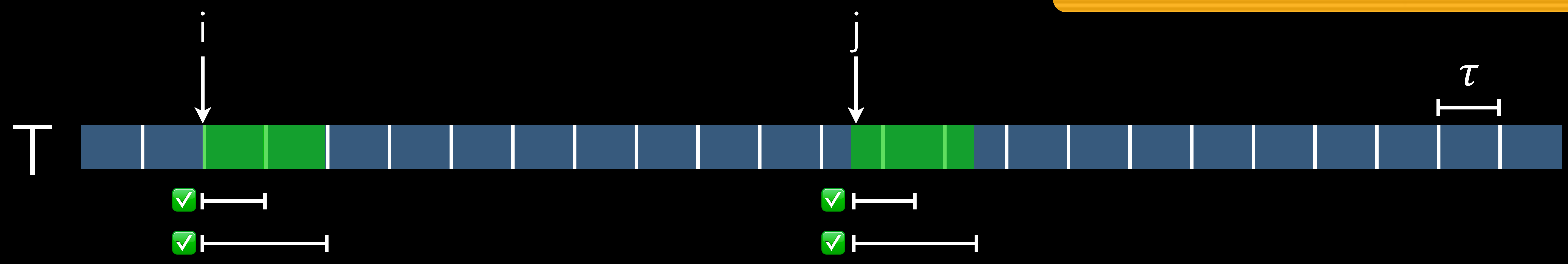

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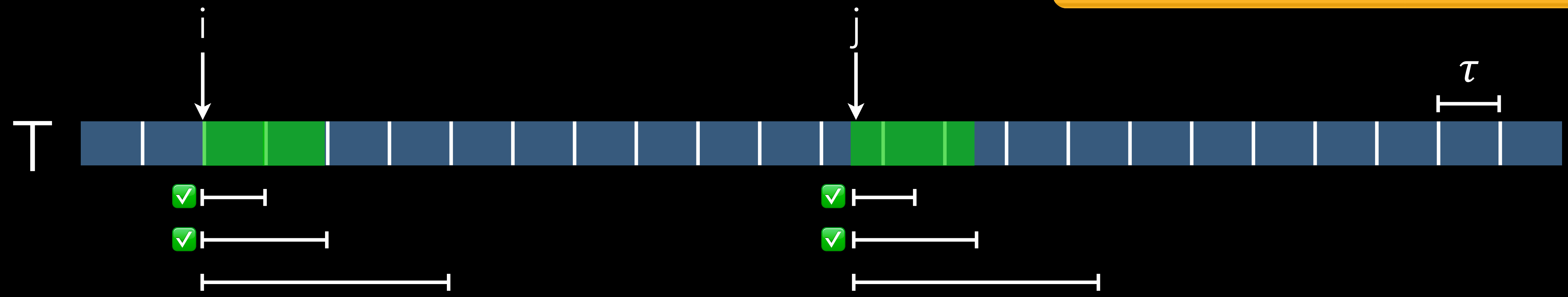

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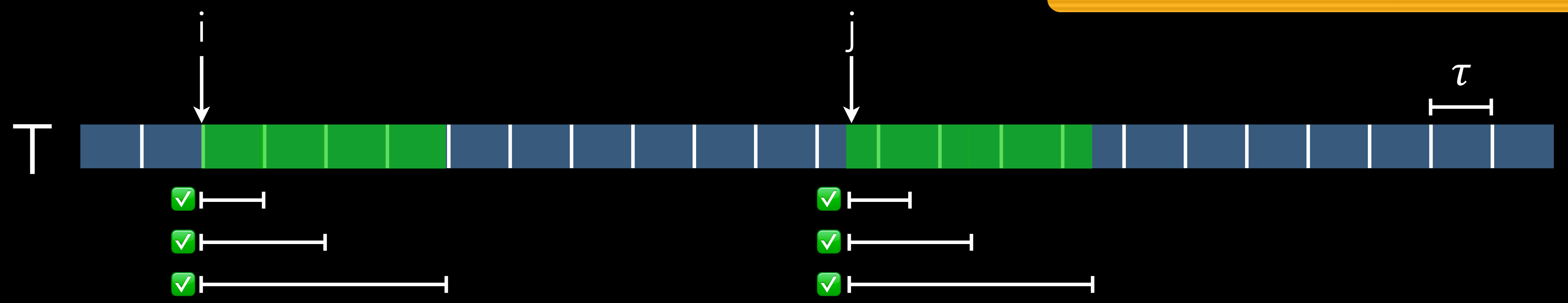

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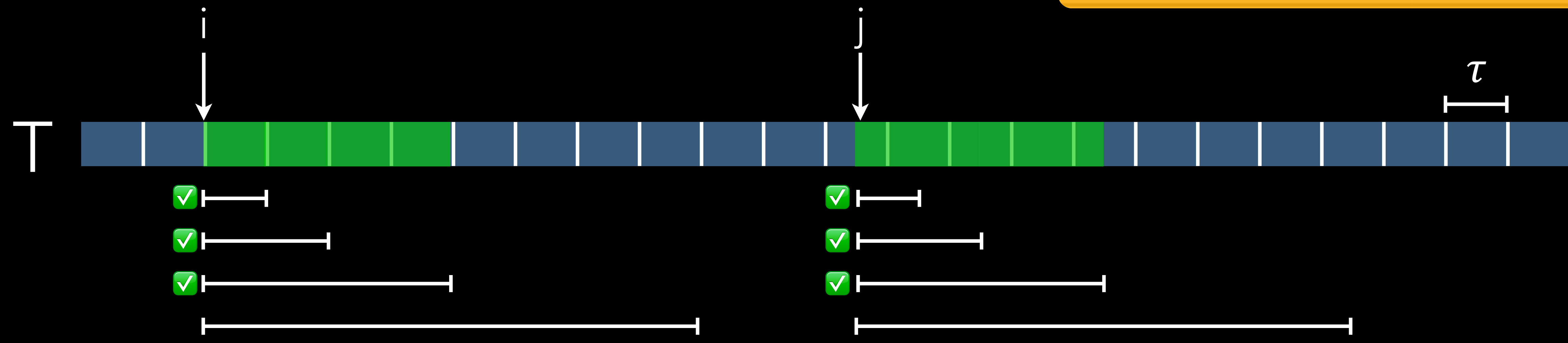

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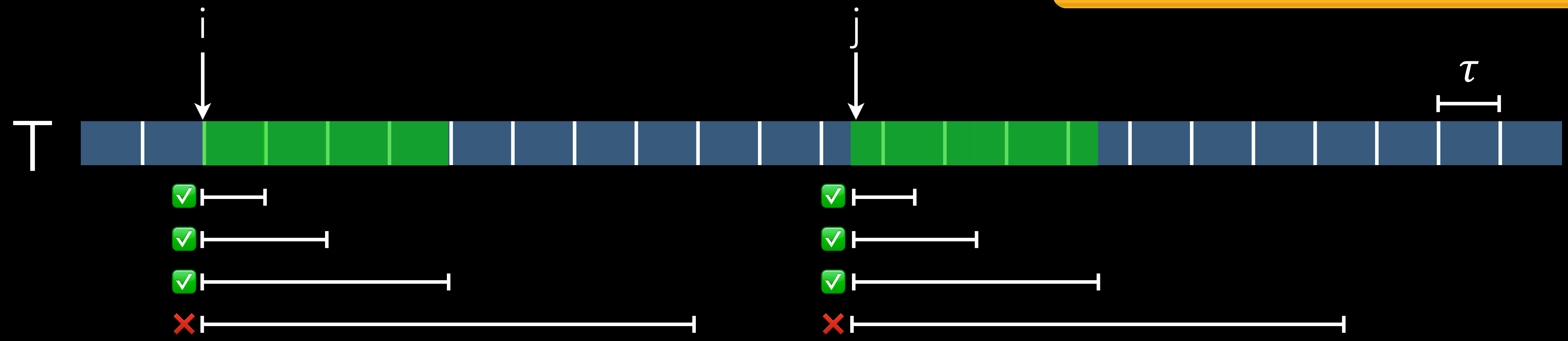

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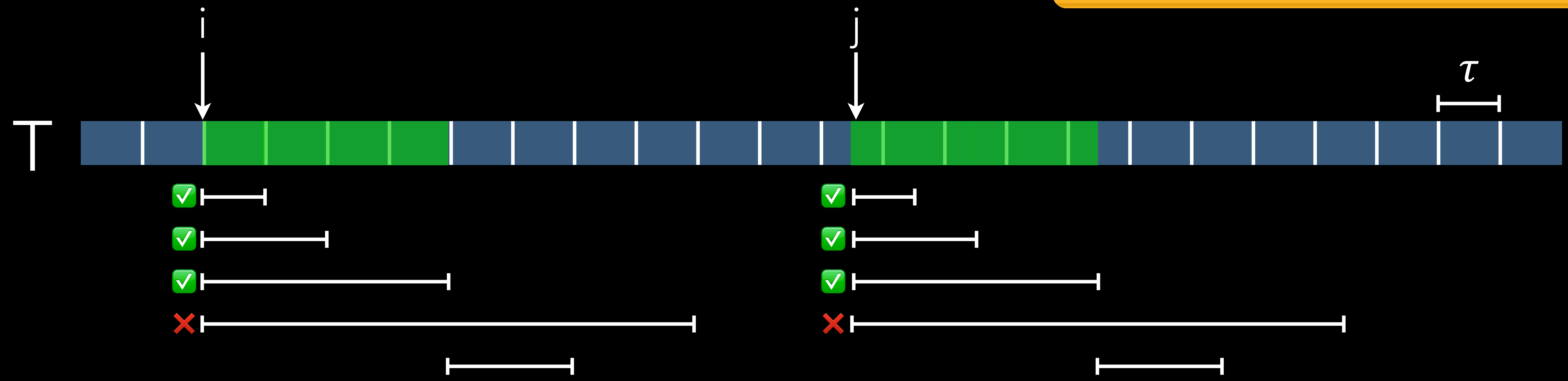

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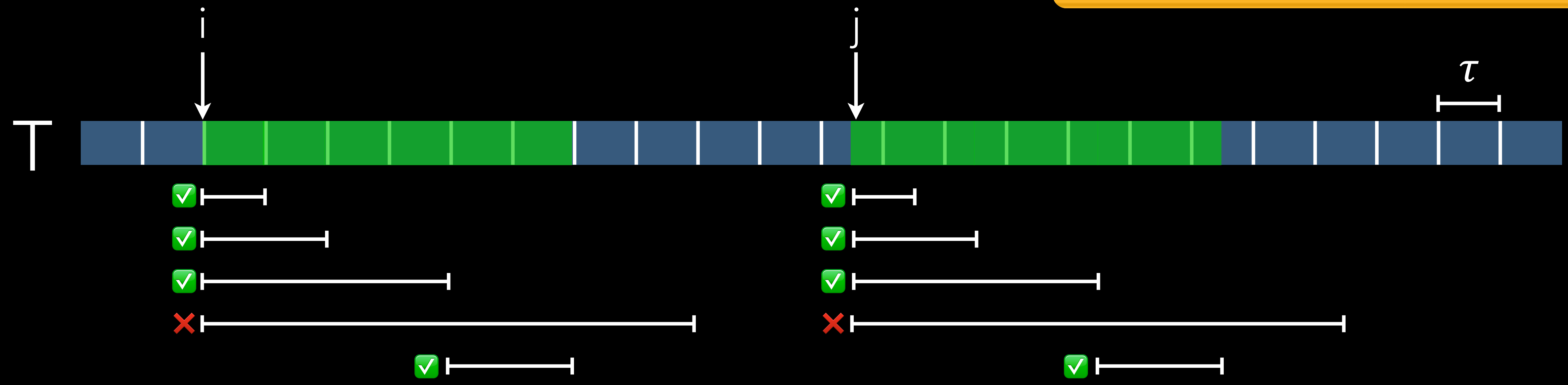

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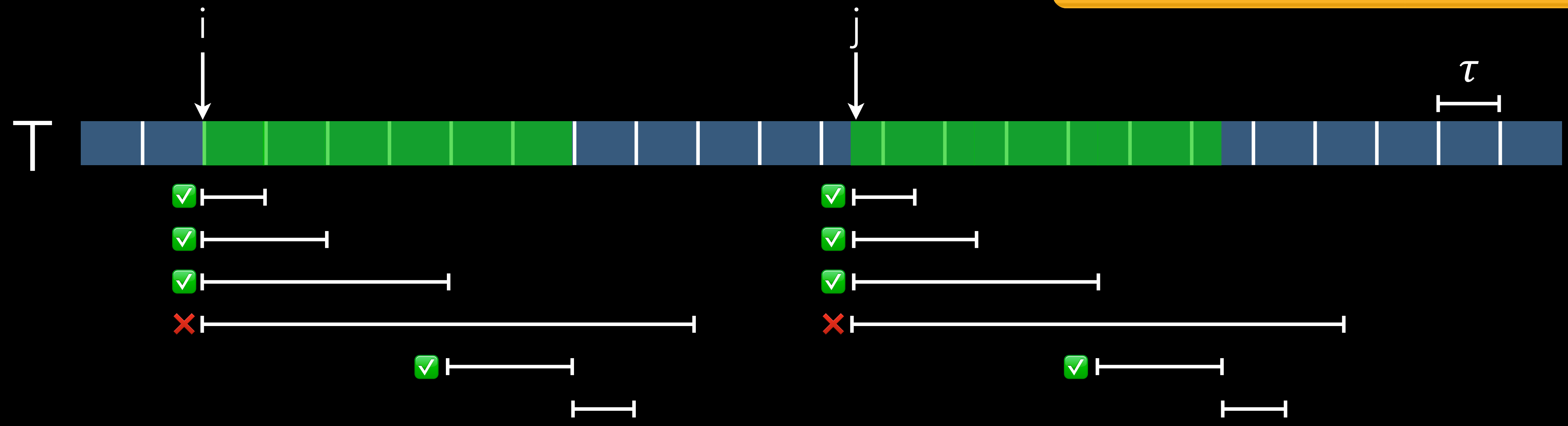

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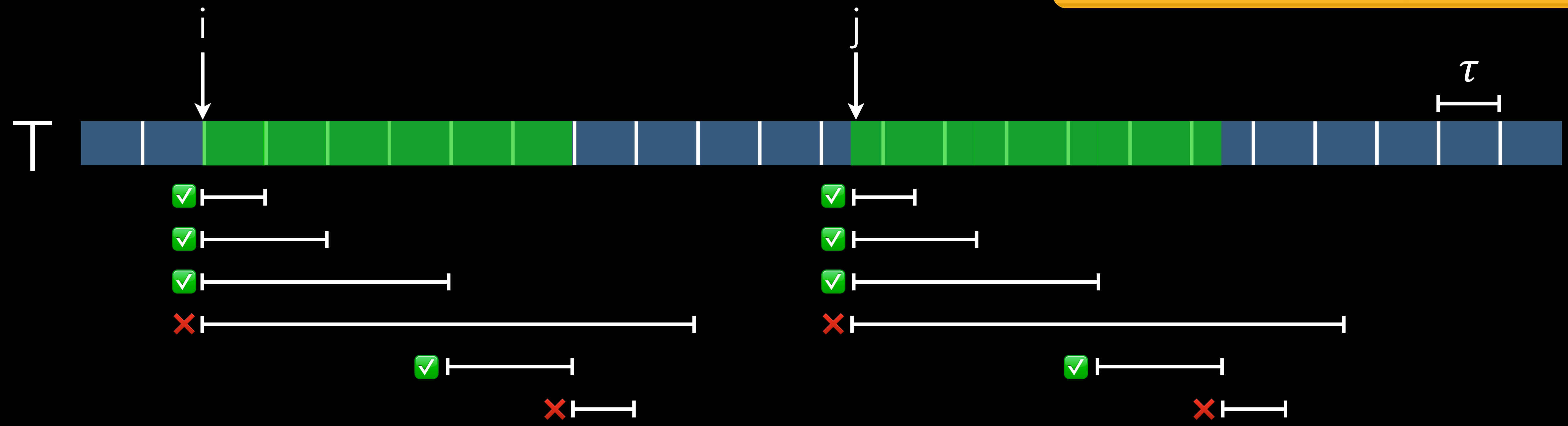
   (Compare the substrings by their Karp-Rabin fingerprints)

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau \log(\ell/\tau))$ time



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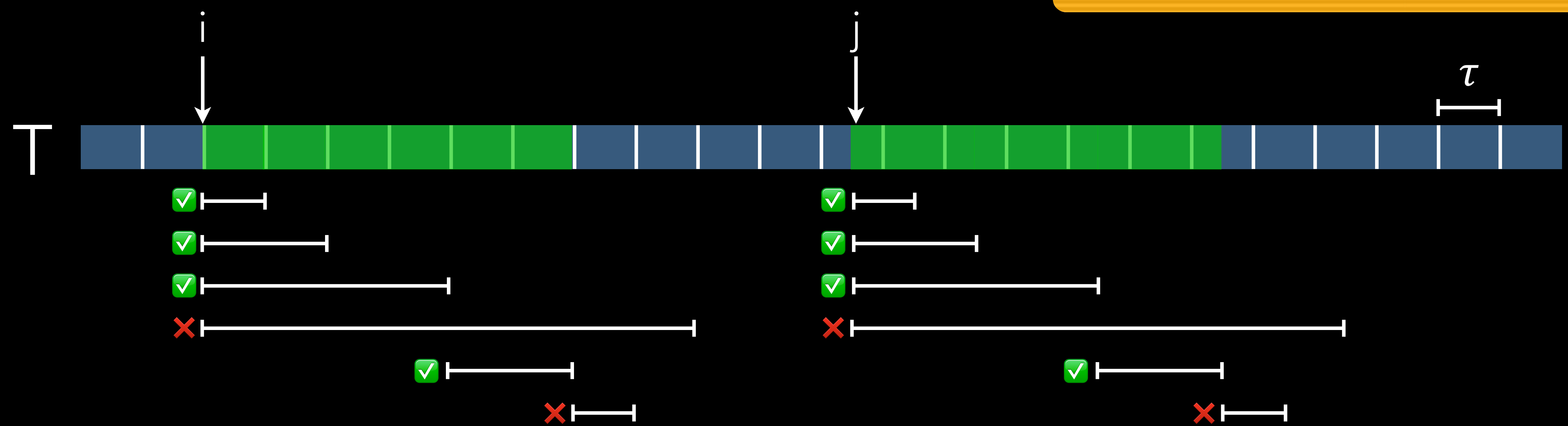

Answering an LCE query

1. Perform exponential search to find an interval containing the first mismatch (Compare the substrings by their Karp-Rabin fingerprints)
2. Scan the interval directly to find the mismatch

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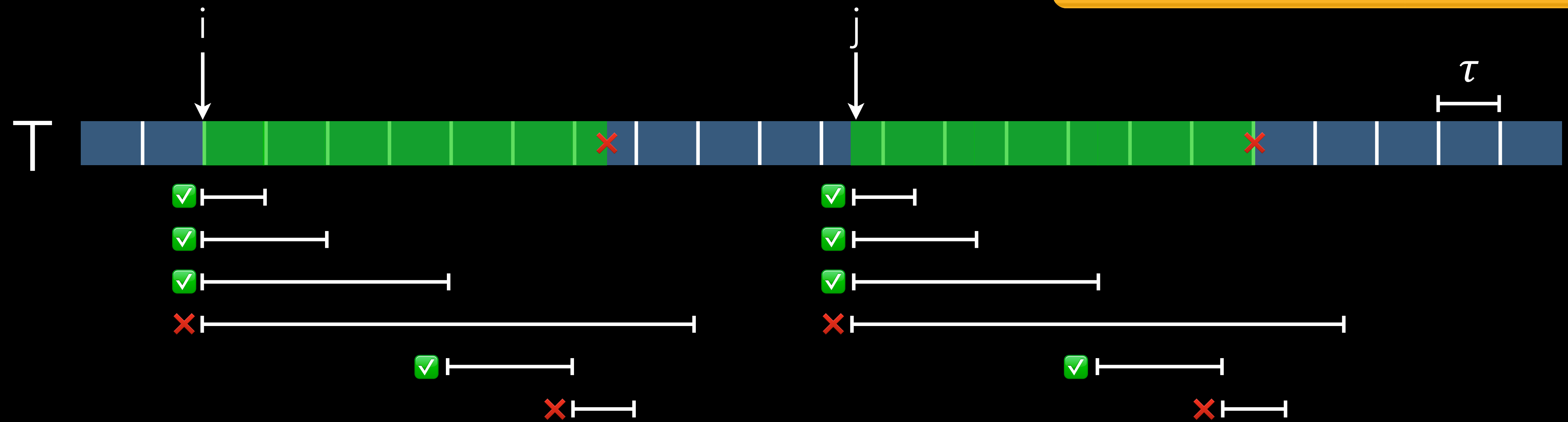

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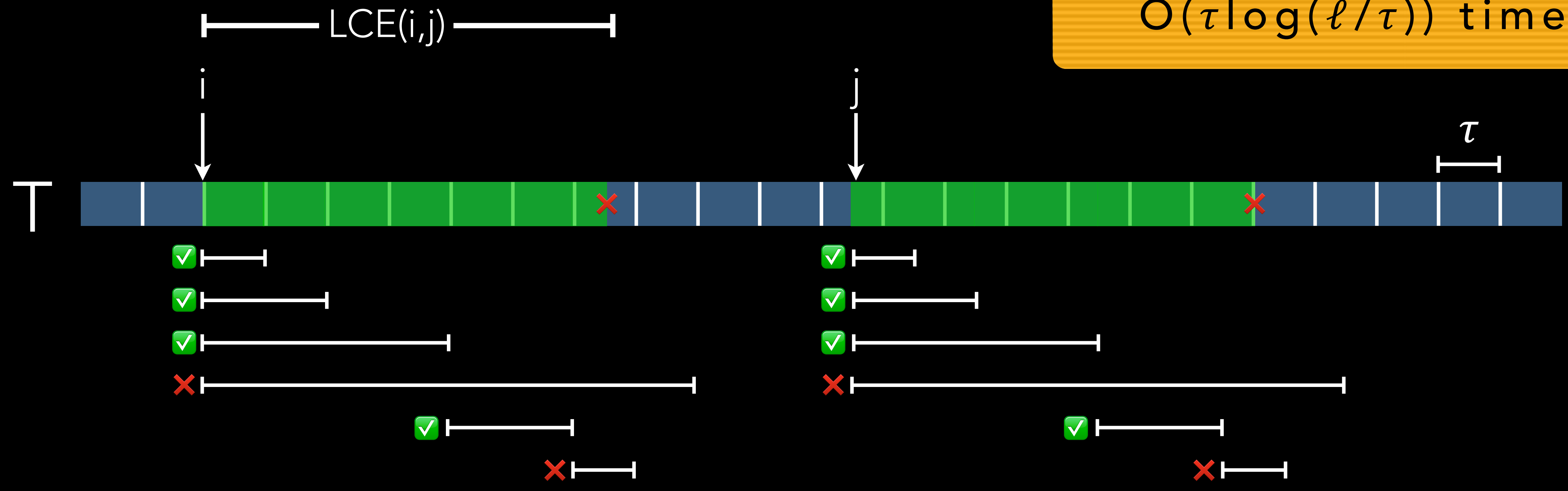

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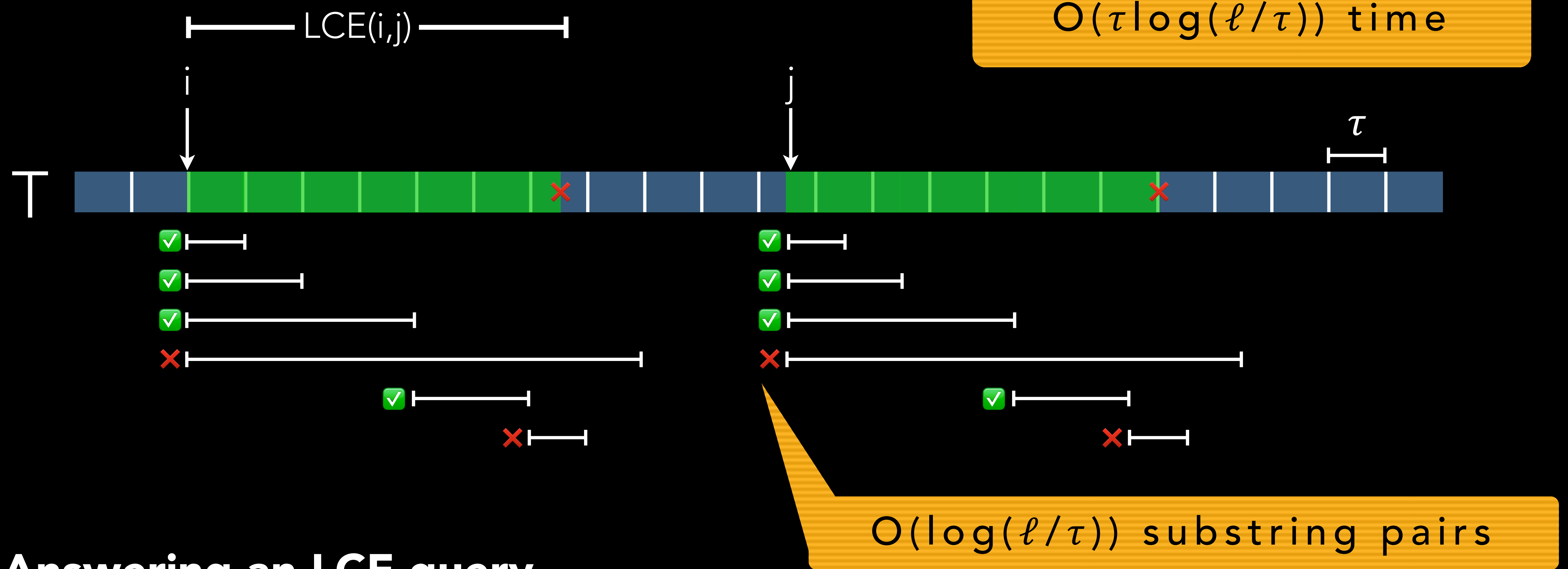

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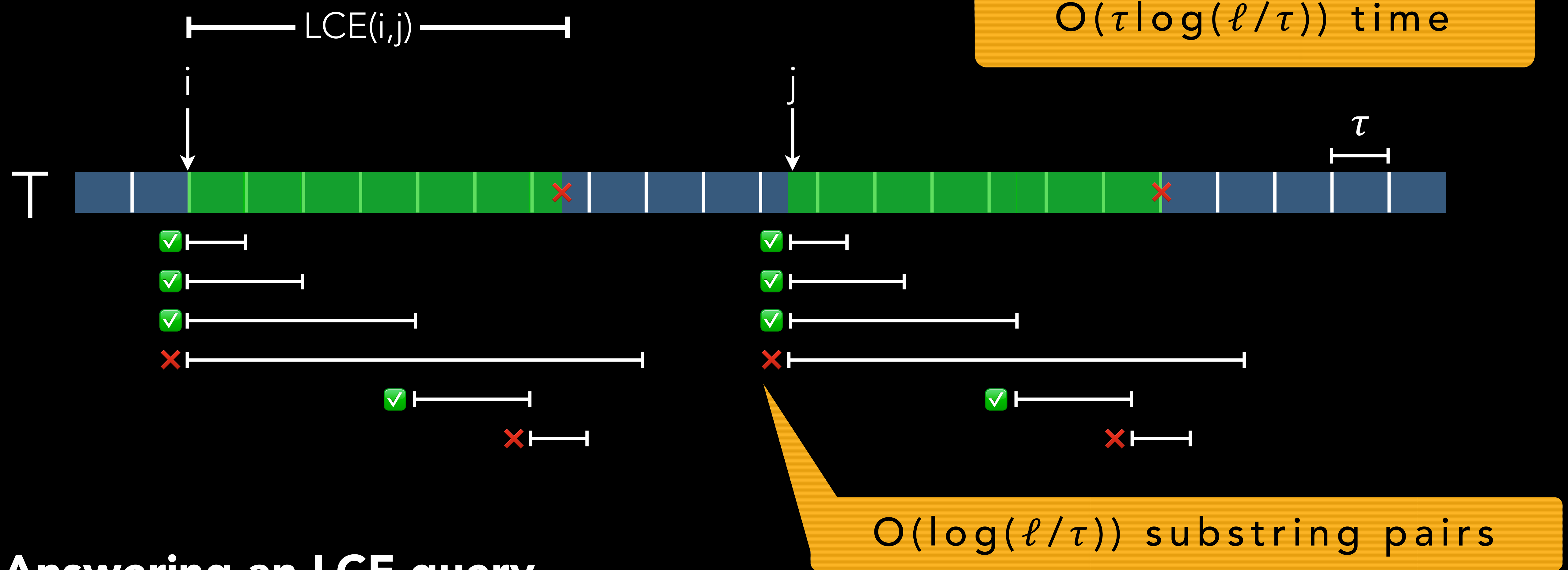

Answering an LCE query

1. Perform exponential search to find an interval containing the first mismatch (Compare the substrings by their Karp-Rabin fingerprints)
2. Scan the interval directly to find the mismatch

Data structure

Stores fingerprint of every block aligned suffix

$\Rightarrow$ the fingerprint of any substring can be retrieved in $O(\tau)$ time

NEXT STEP

$O(n/\tau)$ space
$O(\tau \log^2(n/\tau))$ time ✅

↓

$O(n/\tau)$ space
$O(\tau \log(\ell/\tau))$ time ✅

↓

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

↓

$O(n/\tau)$ space
$O(\tau)$ time

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

Some definitions

T

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

Some definitions

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

Some definitions

$$q = 205035 = \underbrace{\overbrace{0\,1\,1\,0\,0\,1\,0\,0\,0\,0\,0}^{\lg(n/\tau)}}_{\text{head}}\ \underbrace{\overbrace{1\,1\,1\,0\,1\,0\,1\,1}^{\lg(\tau)}}_{\text{tail}}$$

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau+\log(\ell/\tau))$ time

Some definitions

$$q = 205035 = \overbrace{0\,1\,1\,0\,0\,1\,\underbrace{0\,0\,0\,0\,0}_{\text{significance}}}^{\lg(n/\tau)}\,\overbrace{1\,1\,1\,0\,1\,0\,1\,1}^{\lg(\tau)}$$

head significance tail

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

Some definitions

$$\mathbf{q} = \mathbf{205035} = \underbrace{\overbrace{\mathbf{011001}}^{\text{head}}\ \overbrace{\mathbf{00000}}^{\text{significance}}}_{\lg(n/\tau)}\ \underbrace{\mathbf{11101011}}_{\lg(\tau)\ \text{tail}}$$

In a block k we *sample* b_k evenly spaced positions, where $b_k = \min(2^{\mu/2}, \tau)$

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

Some definitions

$\lg(n/\tau)$ $\lg(\tau)$

q = 205035 = 0 1 1 0 0 1 0 0 0 0 0 1 1 1 0 1 0 1 1

head **significance** **tail**

In a block k we *sample* b_k evenly spaced positions, where $b_k = \min(2^{\mu/2}, \tau)$

Significance of block k

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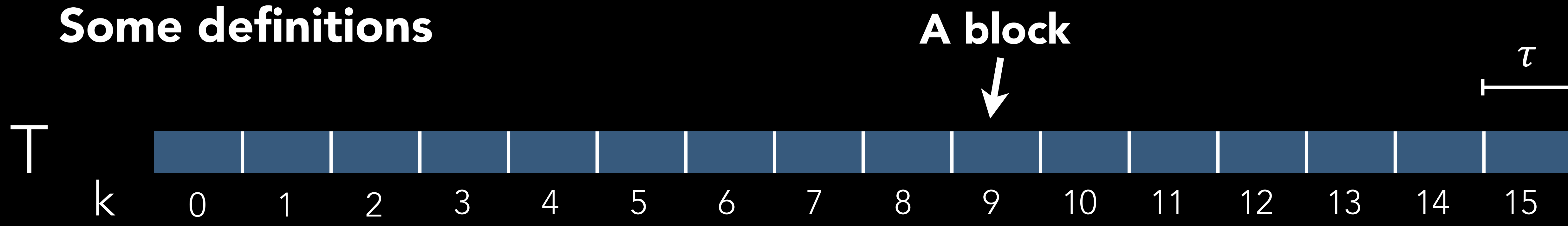

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Some definitions

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
μ	4	0	1	0	2	0	1	0	3	0	1	0	1	0	1	0

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Significance of block k

RANDOMIZED TRADE-OFF

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Some definitions

A block

τ

T

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μ	4	0	1	0	2	0	1	0	3	0	1	0	1	0	1	0
b_k	4	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1

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In a block k we *sample* b_k evenly spaced positions, where $b_k = \min(2^{\mu/2}, \tau)$

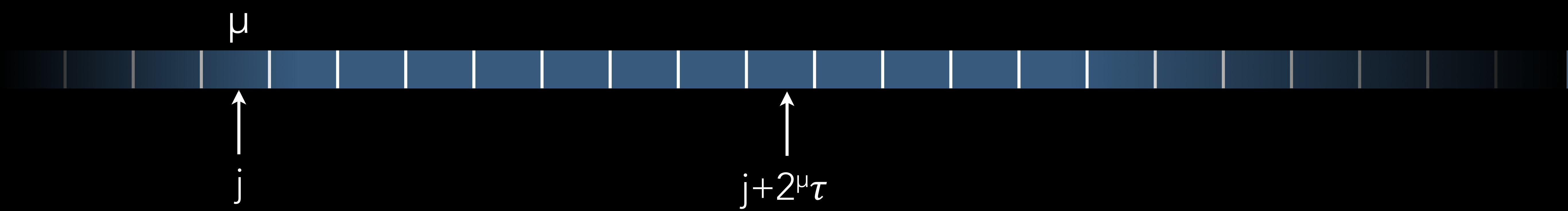
Significance of block k

Bounding the number of sampled positions

$$|\mathcal{S}| = \sum_{k=0}^{n/\tau-1} b_k \leq \sum_{\mu=0}^{\lg(n/\tau)} 2^{\lg(n/\tau)-\mu} 2^{\lfloor \mu/2 \rfloor} \leq \frac{n}{\tau} \sum_{\mu=0}^{\infty} 2^{-\mu/2} = \left(2+\sqrt{2}\right)\frac{n}{\tau} = O\left(\frac{n}{\tau}\right)$$

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time



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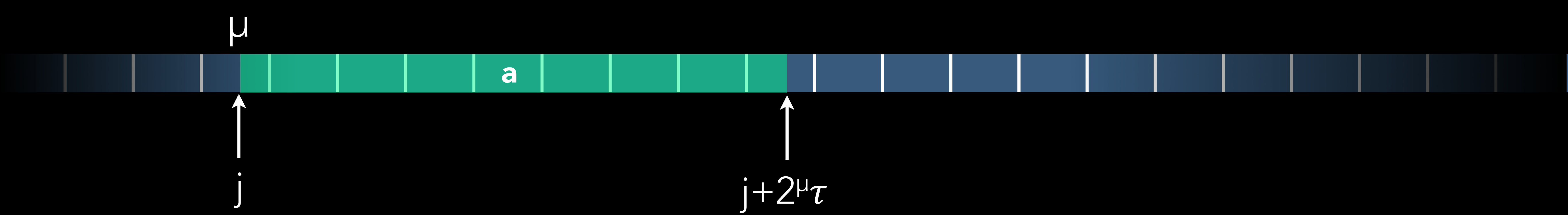

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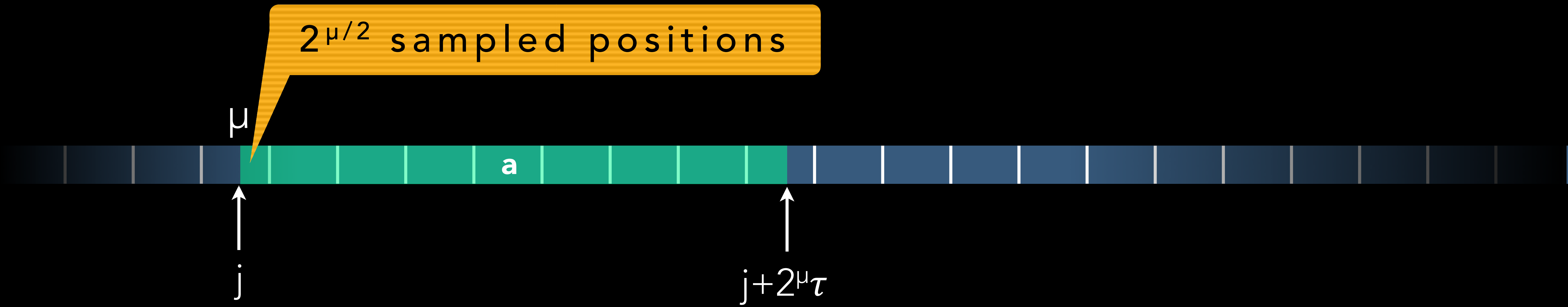
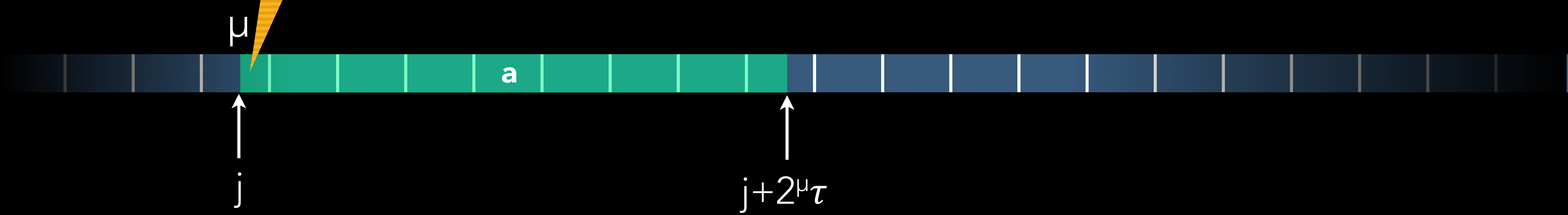


RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau + \log(\ell/\tau))$ time

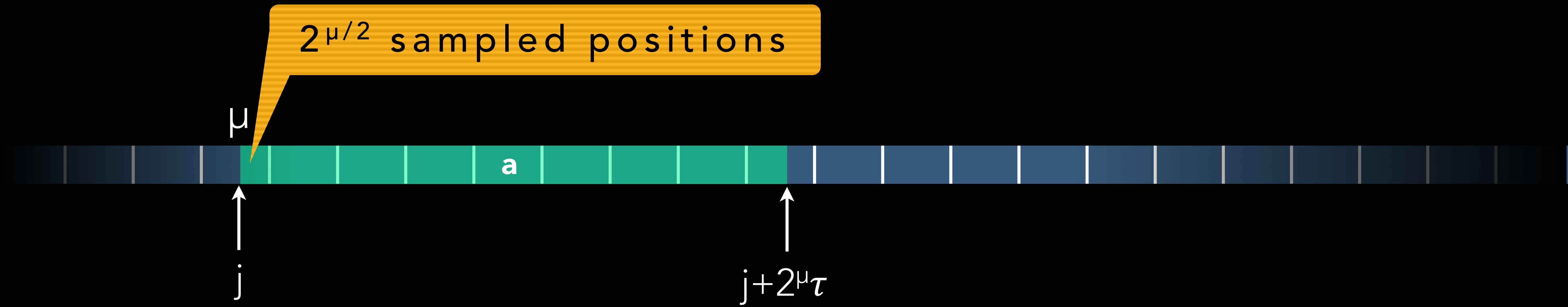
$2^{\mu/2}$ sampled positions



$$j = \dots 1\overbrace{0\dots\dots 0}^{\mu}[\text{tail}]$$

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
$O(\tau+\log(\ell/\tau))$ time



$$j = \ldots.1\overbrace{0\ldots\ldots0}^{\mu}[\text{tail}]$$

$$2^{\mu}\tau = \ldots.10\ldots\ldots0000..$$

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$O(n/\tau)$ space
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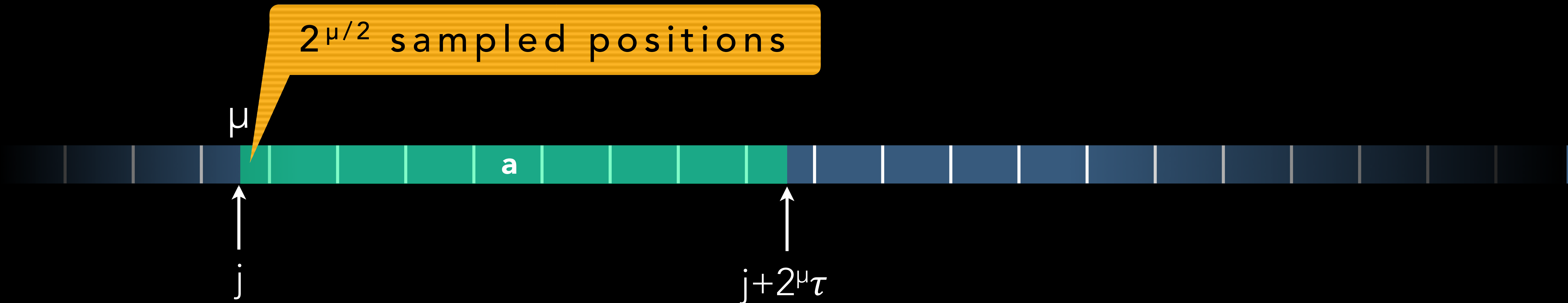

$$j = \ldots.1\overbrace{0\ldots..0}^{\mu}[\mathrm{tail}]$$

$$2^{\mu}\tau = \ldots.10\ldots..0000..$$

$$j+2^{\mu}\tau = \ldots.00\ldots..0[\mathrm{tail}]$$

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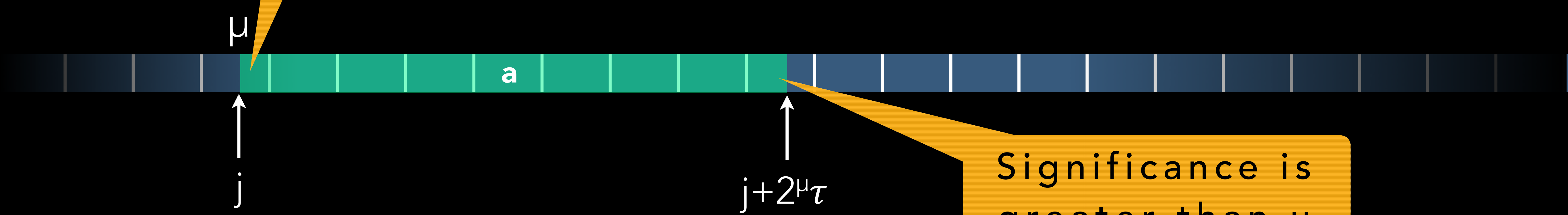
$2^{\mu/2}$ sampled positions



Significance is greater than μ

$$j = \dots.1\overbrace{0\dots\dots}^{\mu}0[\text{tail}]$$

$$2^{\mu}\tau = \dots.10\dots\dots0000..$$

$$j+2^{\mu}\tau = \dots.00\dots\dots0[\text{tail}]$$

RANDOMIZED TRADE-OFF

$O(n/\tau)$ space
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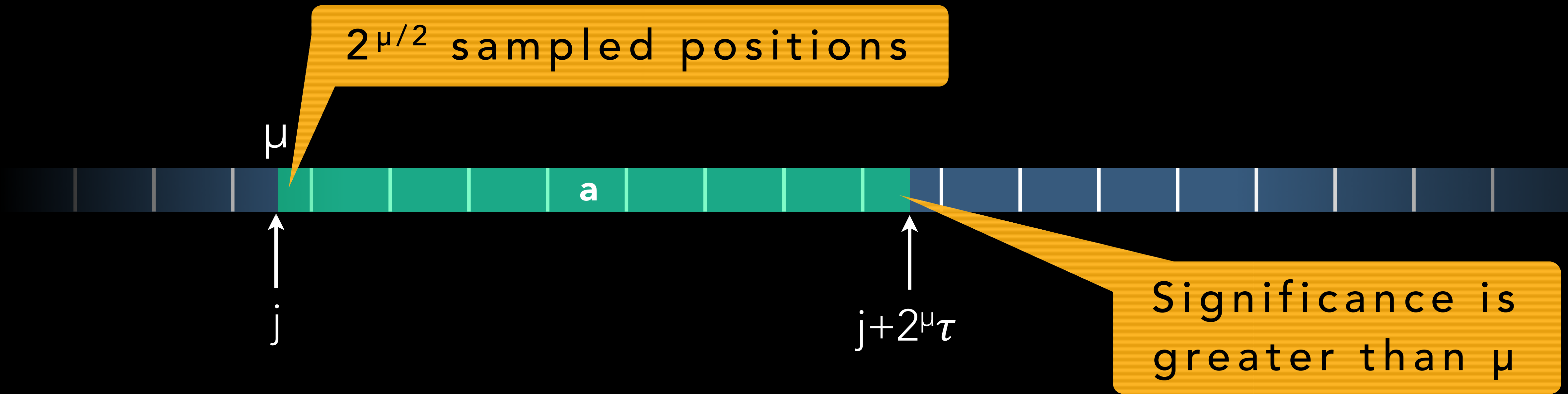

$$j = \ldots 1\overbrace{0\ldots\ldots 0}^{\mu}[\text{tail}]$$

$$2^{\mu}\tau = \ldots 10\ldots\ldots 0000\ldots$$

$$j+2^{\mu}\tau = \ldots 00\ldots\ldots 0[\text{tail}]$$

Distance to a sampled position is at most $\tau/2^{\mu/2}$

RANDOMIZED TRADE-OFF

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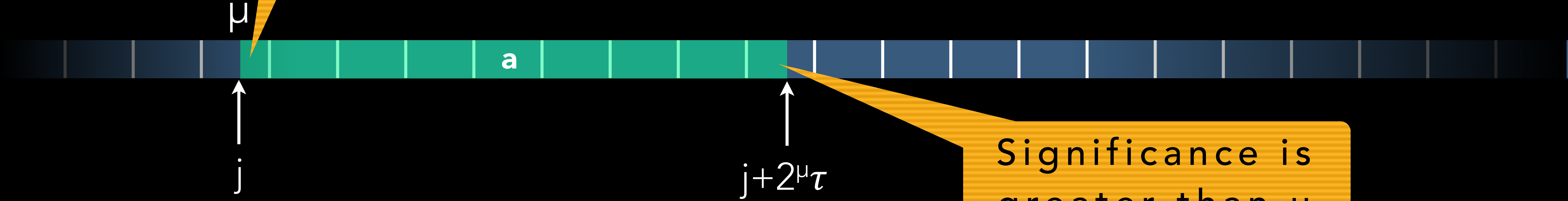
$O(\tau+\log(\ell/\tau))$ time

$2^{\mu/2}$ sampled positions



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$$j+2^{\mu}\tau = \ldots.00\ldots..0[\text{tail}]$$

Distance to a sampled position is at most $\tau/2^{\mu/2}$

$\Longrightarrow$ Time to compute $\varphi(a)$ is $O(1+\tau/2^{\mu/2})$

RANDOMIZED TRADE-OFF

Analysis

$O(n/\tau)$ space
$O(\tau+\log(\ell/\tau))$ time

Query time

Cost of computing a fingerprint is $O(1+\tau/2^{\mu/2})$, and μ iterates from 0 to $\log(\ell/\tau)$ and back to 0, thus the query time becomes

$$O\left(\sum_{\mu=0}^{\lg(\ell/\tau)} 1+\tau/2^{\lfloor\mu/2\rfloor}\right) = O(\tau+\log(\ell/\tau))$$

Space

Cost is the total number of sampled positions/fingerprints

$$|\mathcal{S}| = \sum_{k=0}^{n/\tau-1} b_k \le \sum_{\mu=0}^{\lg(n/\tau)} 2^{\lg(n/\tau)-\mu}2^{\lfloor\mu/2\rfloor} \le \frac{n}{\tau}\sum_{\mu=0}^{\infty} 2^{-\mu/2} = \left(2+\sqrt{2}\right)\frac{n}{\tau} = O\left(\frac{n}{\tau}\right)$$

NEXT STEP

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$O(\tau\log^2(n/\tau))$ time ✅

↓

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↓

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↓

$O(n/\tau)$ space
$O(\tau)$ time

SOLUTION FOR LONG LCES

$O(n/\tau)$ space
$O(\tau)$ time

Theorem

There is an $O(n/\tau)$ space data structure that in $O(1)$ time either

- **A.** computes the answer to an LCE(i,j) query, or
- **B.** returns a certificate that $\ell < \tau^2$

Observation

In case B the query time of our previous algorithm becomes $O(\tau + \log(\ell/\tau)) = O(\tau)$

Technique

Difference covers

SUMMARY & OPEN PROBLEMS

MAIN THEOREM

> The LCE problem can be solved in $O(n/\tau)$ space and $O(\tau)$ time for all $1 \leq \tau \leq n$

Lower bound from RMQ implies a time-space product of $\Omega(n/\log n)$
Can we close this gap?

Can we obtain optimal preprocessing times?