Sparse Suffix Tree Construction in Small Space

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Benjamin Sach *(University of Warwick)*

THE UNIVERSITY OF WARWICK

The sparse suffix array (SSA)

n

T	b	a	n	a	n	a	s

The sparse suffix array (SSA)

$\longleftarrow n \longrightarrow$

T	b	a	n	a	n	a	s

1	b	a	n	a	n	a	s
2	a	n	a	n	a	s	
3	n	a	n	a	s		
4	a	n	a	s			
5	n	a	s				
6	a	s					
7	s						

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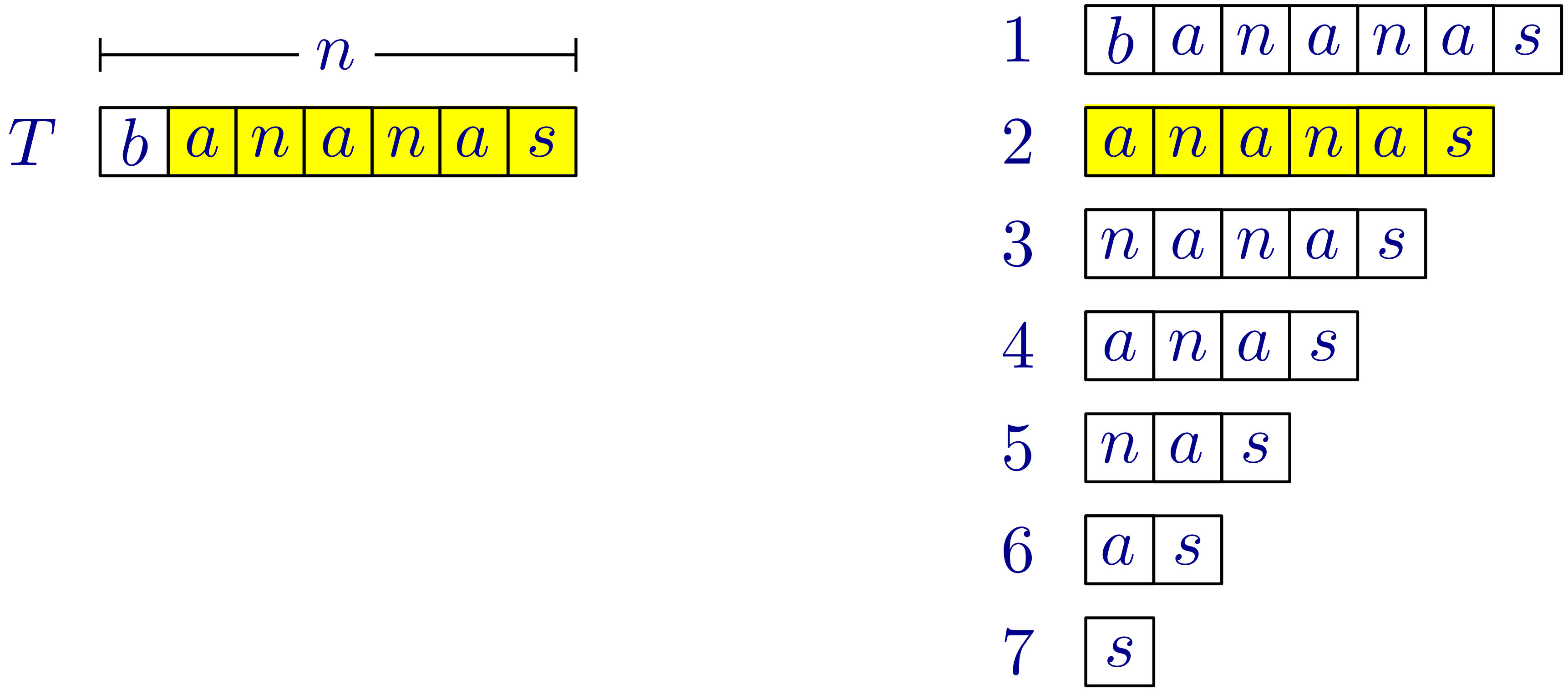

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Sort the suffixes lexicographically

1	b	a	n	a	n	a	s
2	a	n	a	n	a	s	
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Suffix Array	2	4	6	1	3	5	7

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- Can be built in $O(n)$ time and $O(n)$ *extra space*

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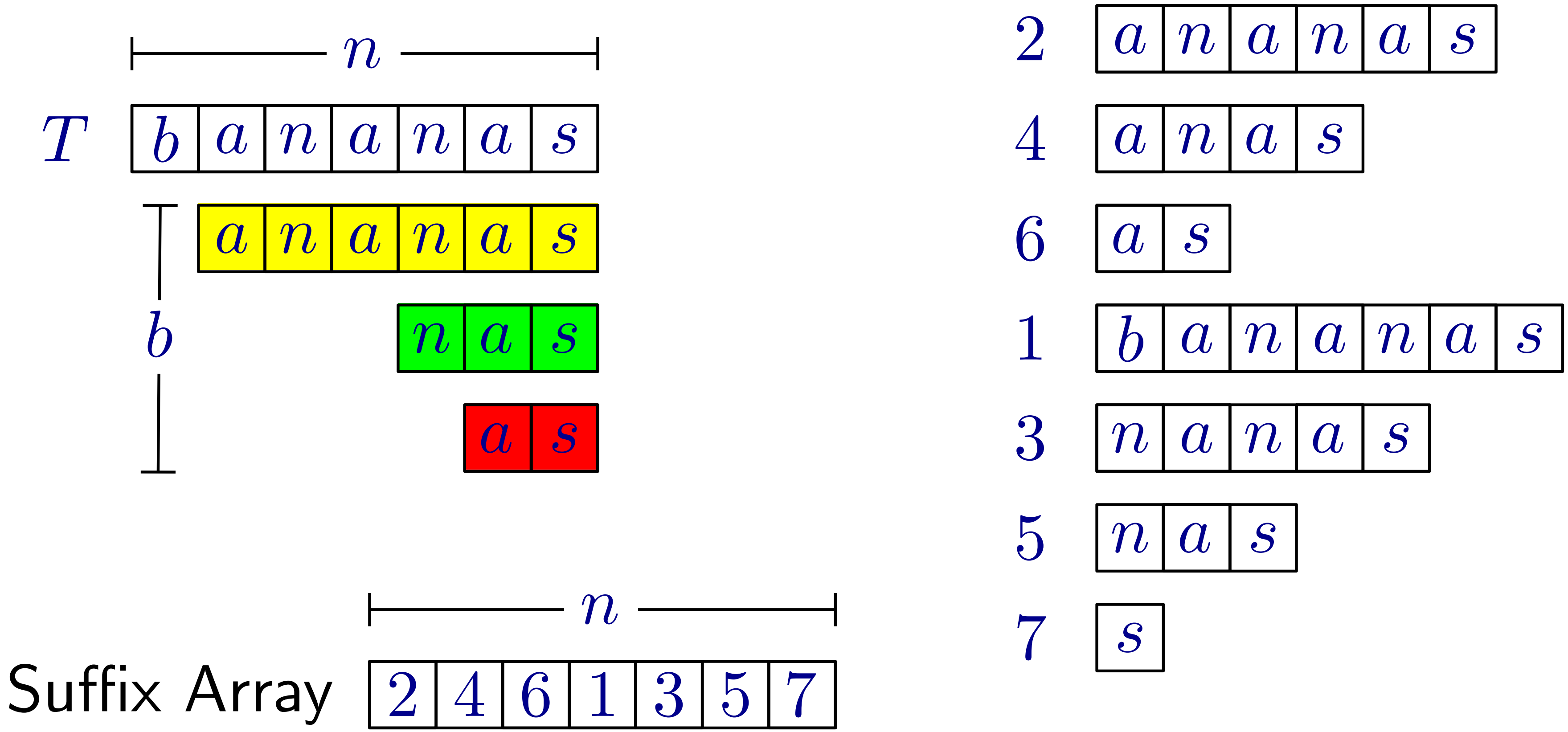

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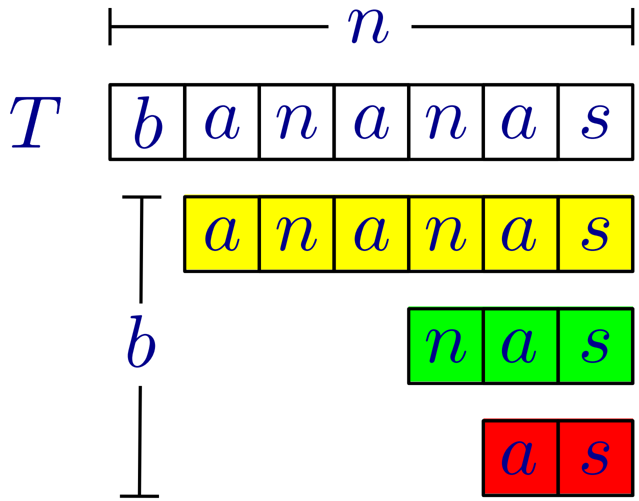

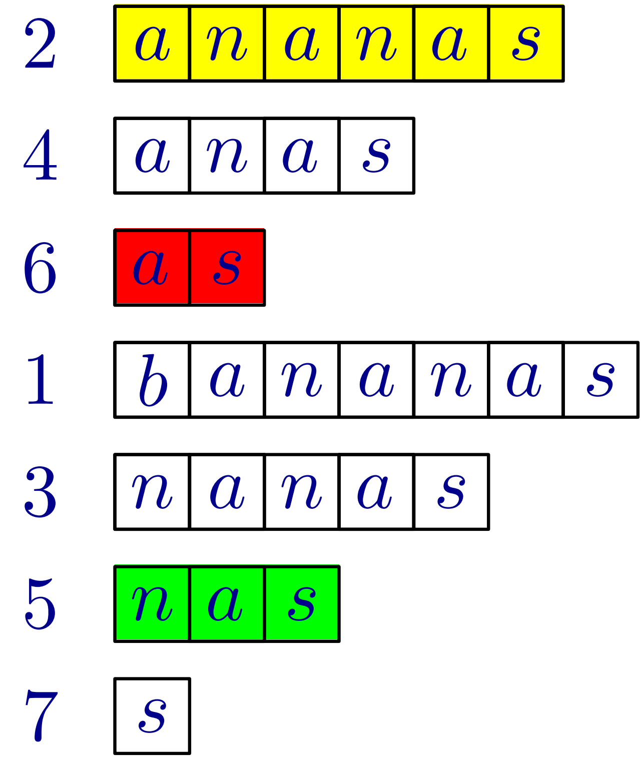

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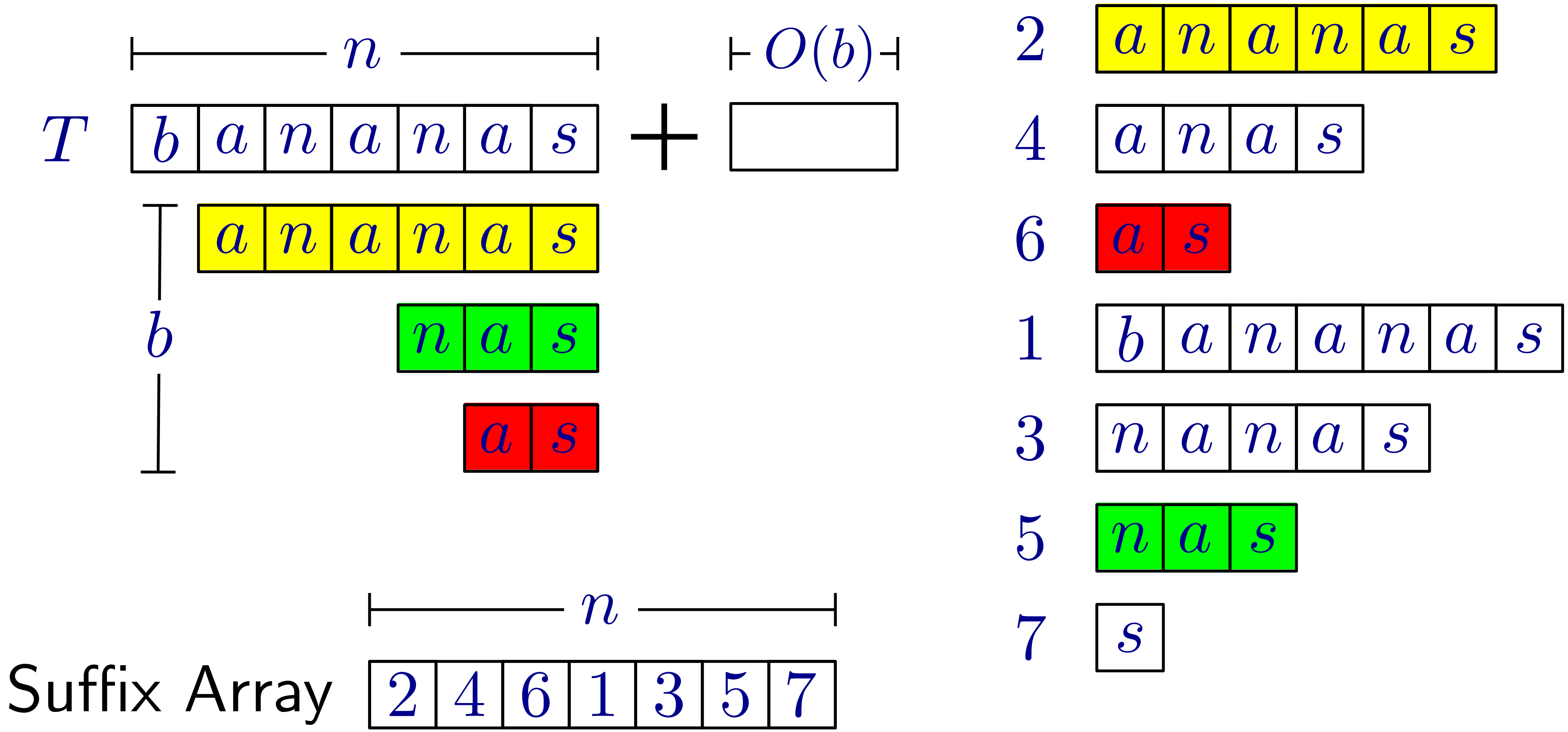

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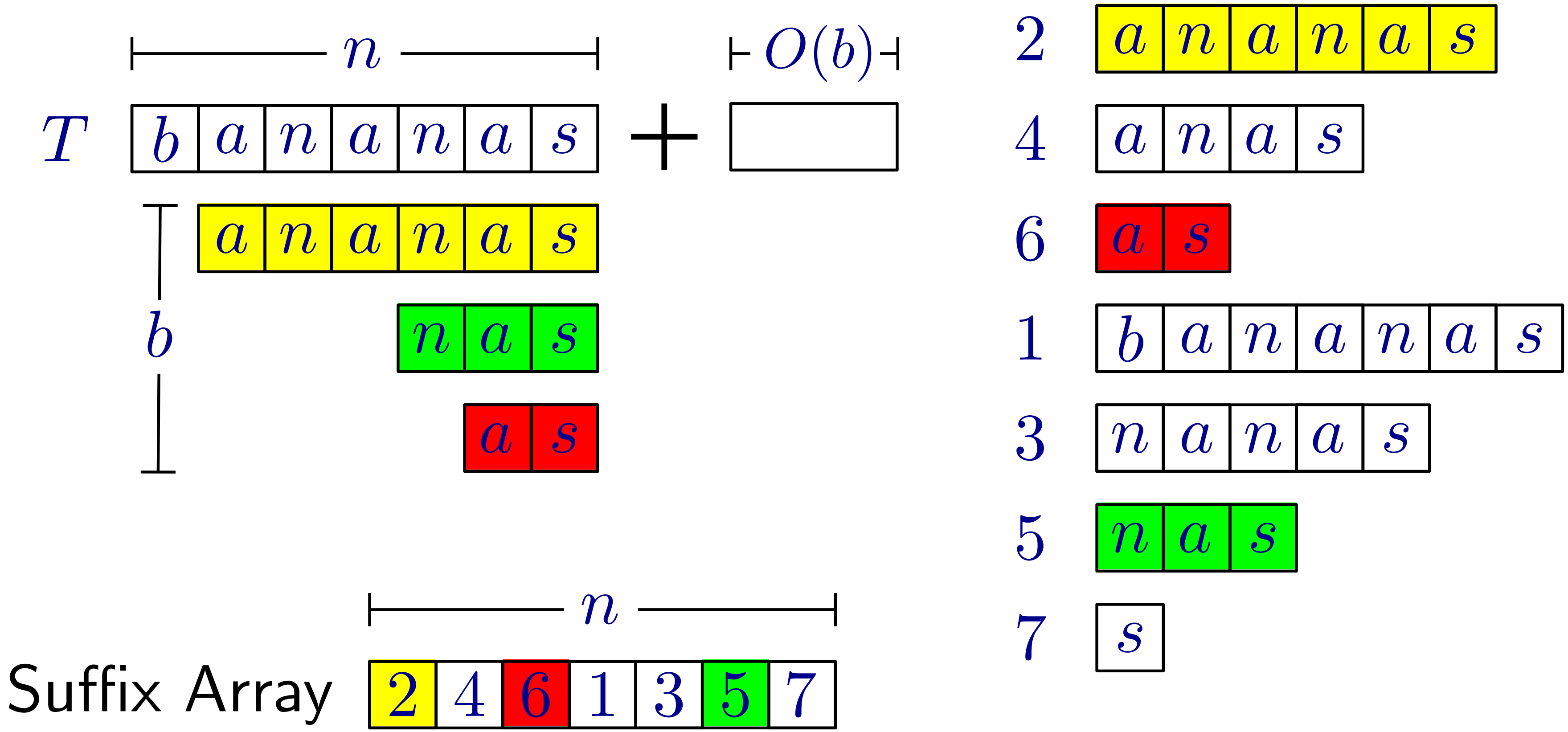

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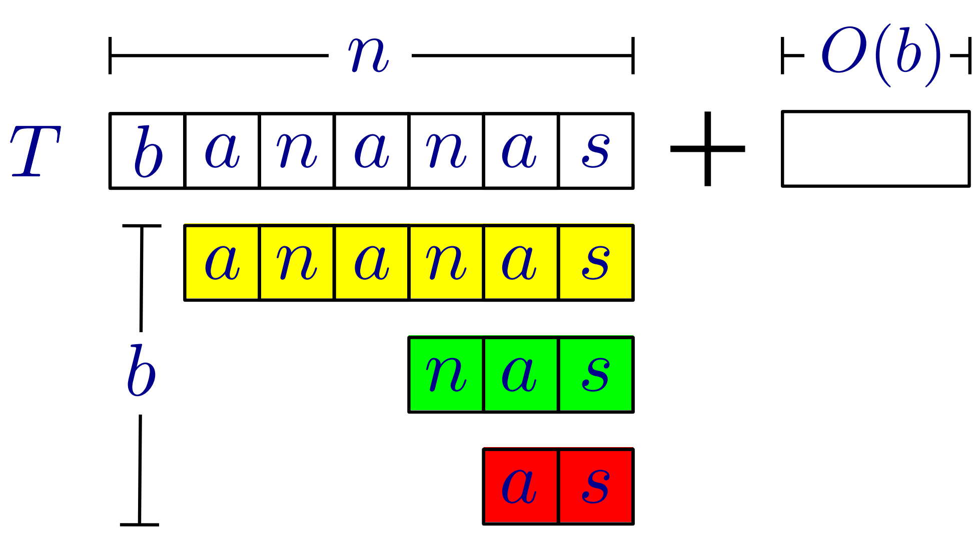

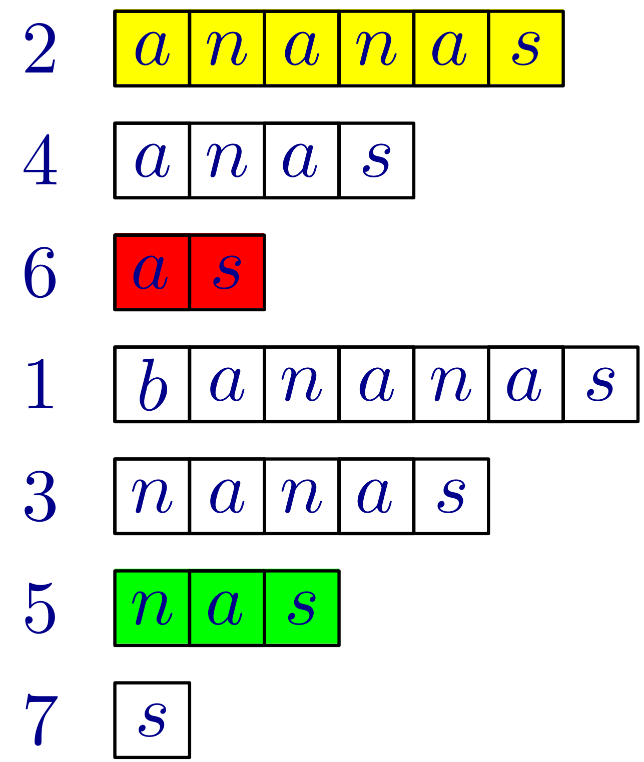

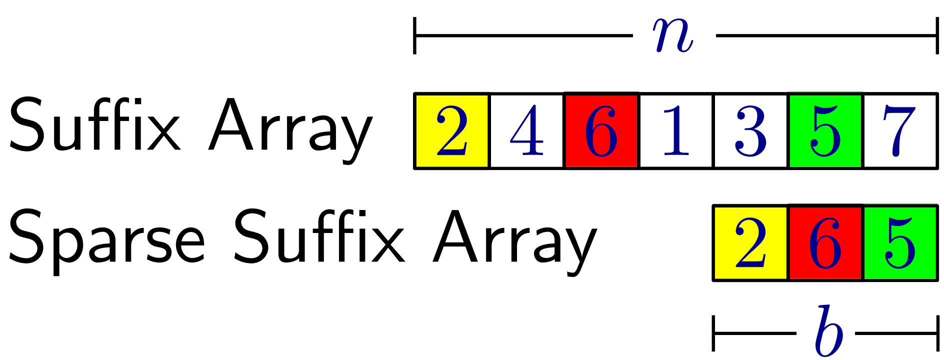

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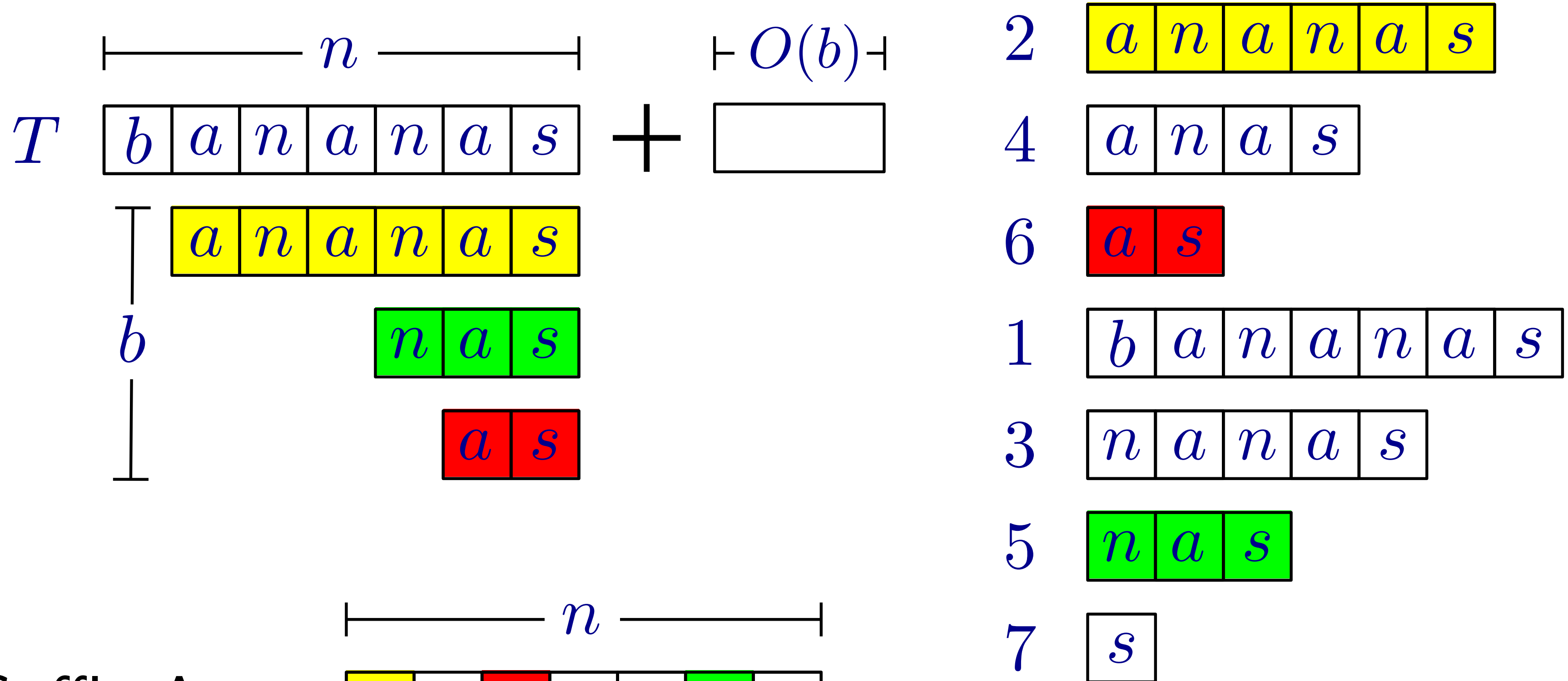

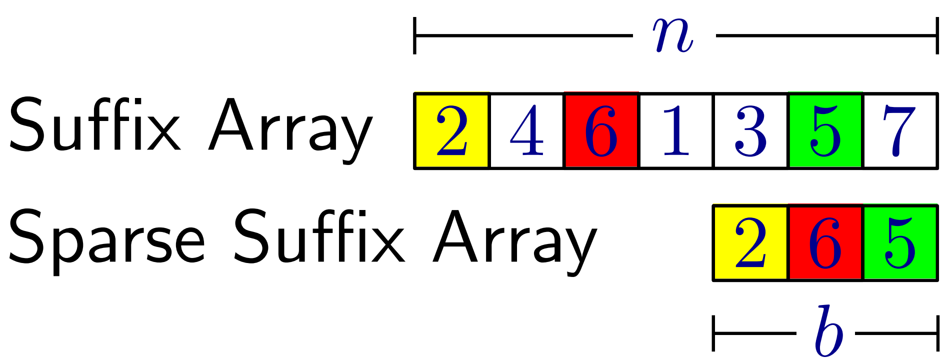

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The sparse text indexing problem has been open since the 1960s
...with first, partial results from 1996 onwards

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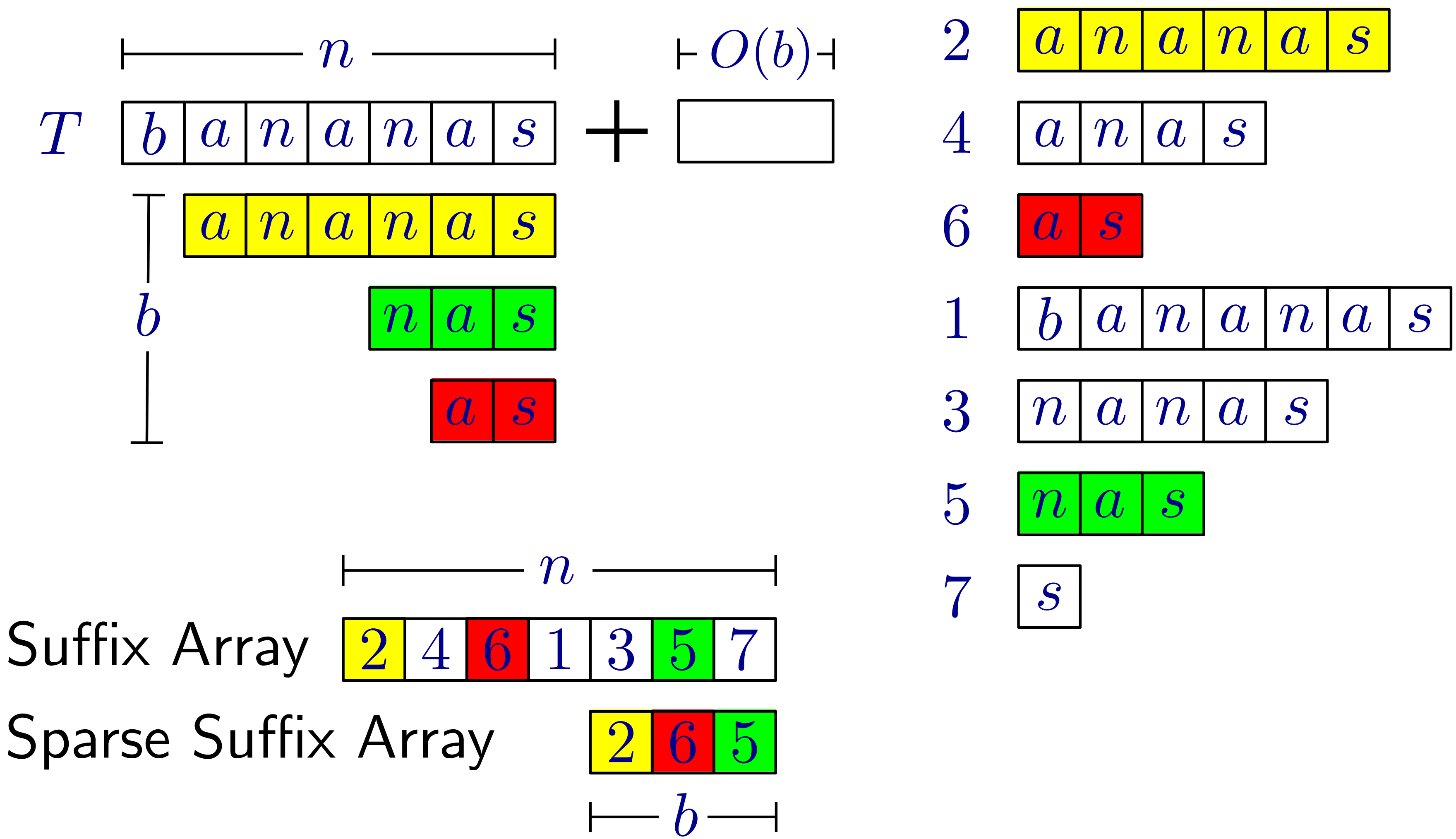

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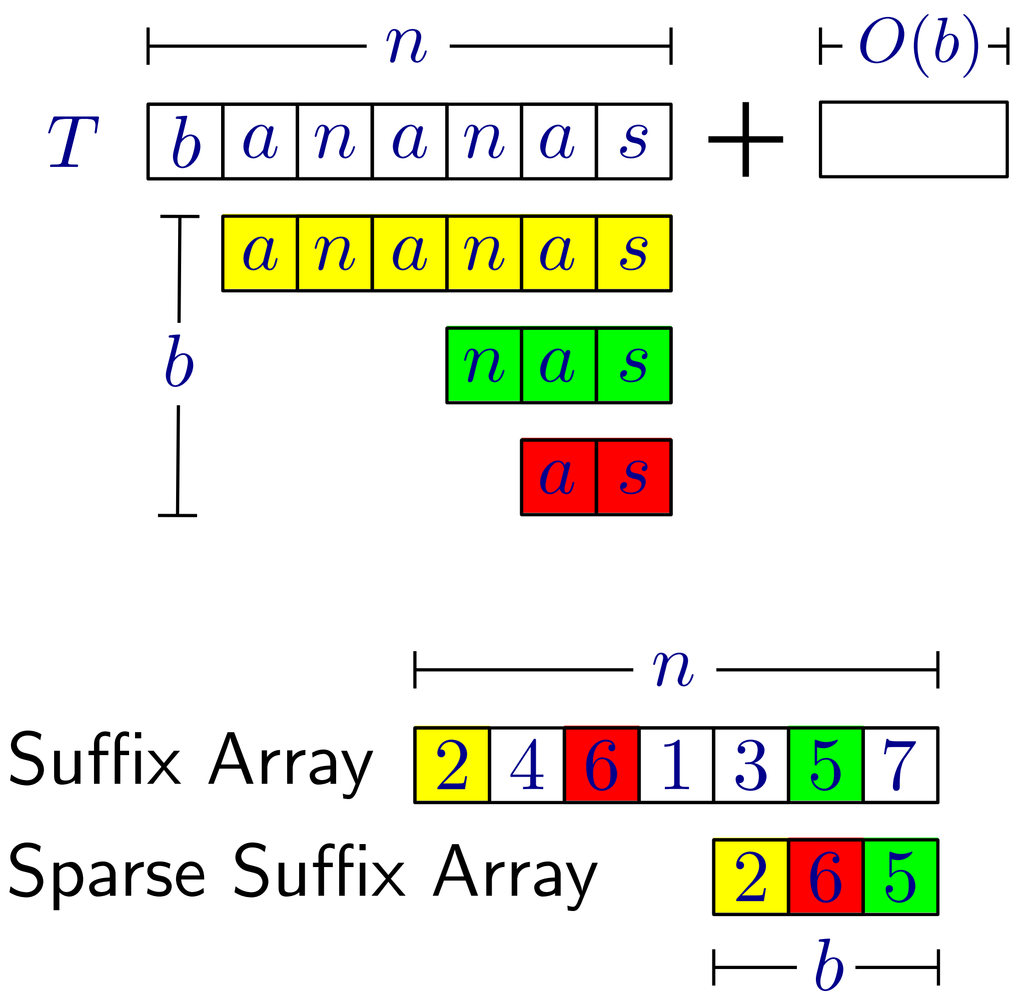
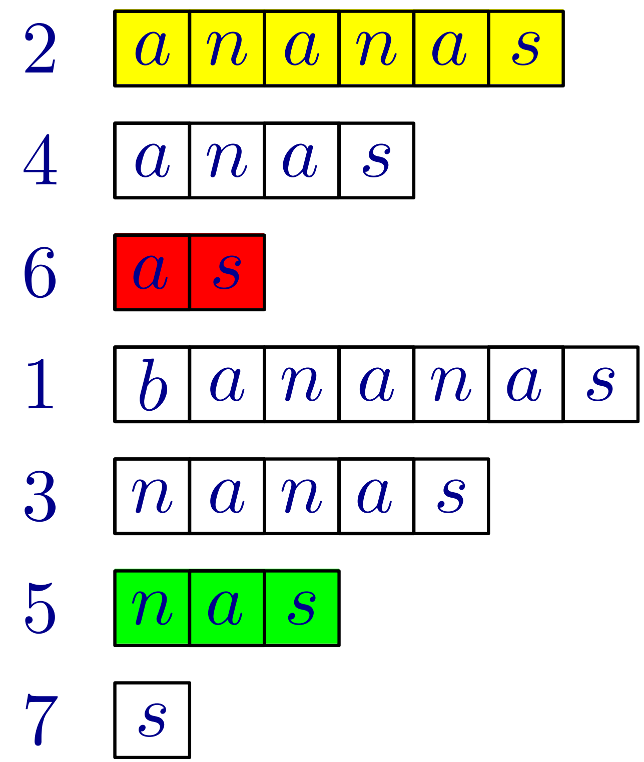
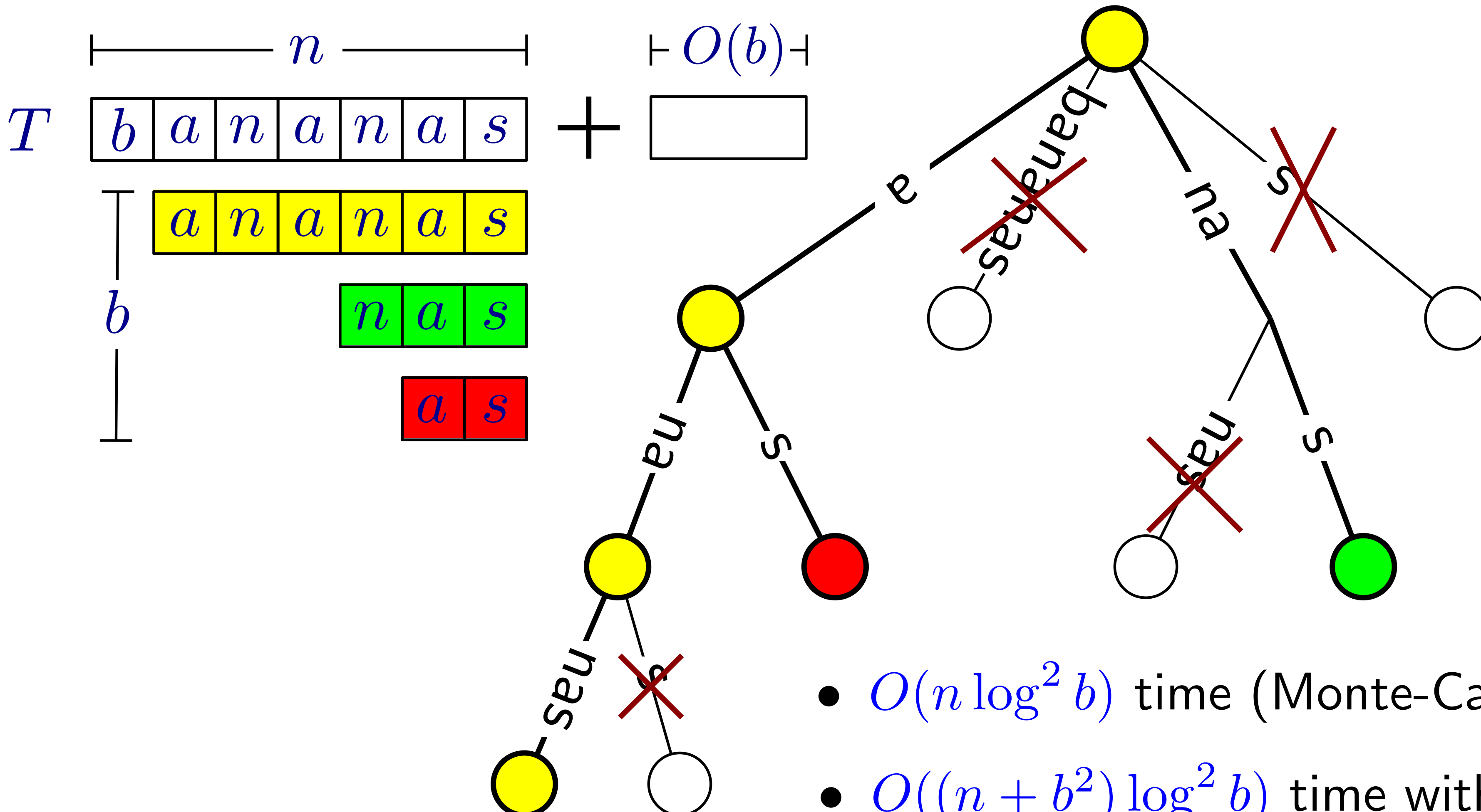




- $O(n \log^2 b)$ time (Monte-Carlo)
- $O((n + b^2) \log^2 b)$ time with high probability (Las-Vegas)
- both in $O(b)$ extra space

The sparse suffix tree (SST)



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Conversion between SSA and SST is simple and takes $O(n \log b)$ *time*

LCPs - a fundamental tool for string algorithms

$$\overbrace{\hspace{20em}}^{n}$$

T	a	b	c	b	a	b	a	b	c	a	b	a	b	a

For any (i, j), the longest common prefix is the largest ℓ such that

$$T[i \ldots i + \ell - 1] = T[j \ldots j + \ell - 1]$$

it's the furthest you can go before hitting a mismatch

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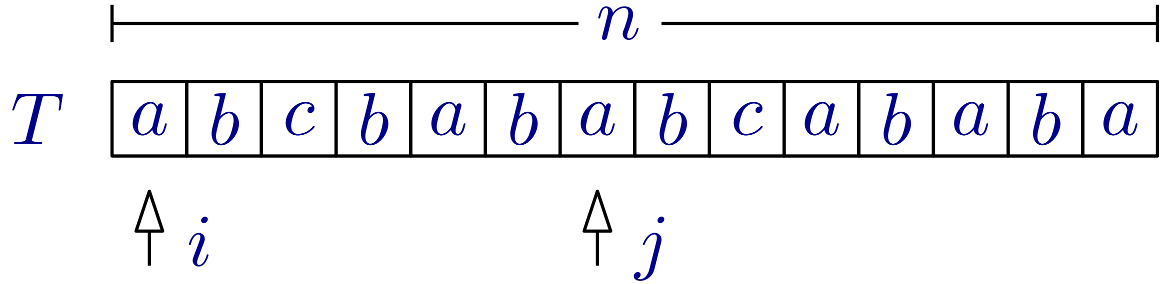

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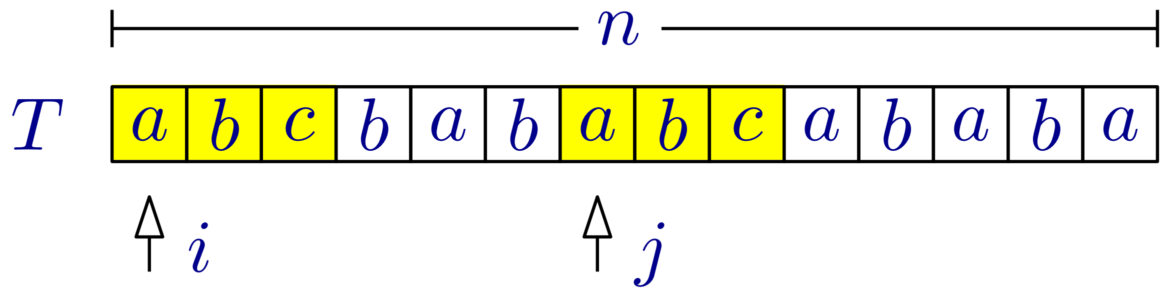

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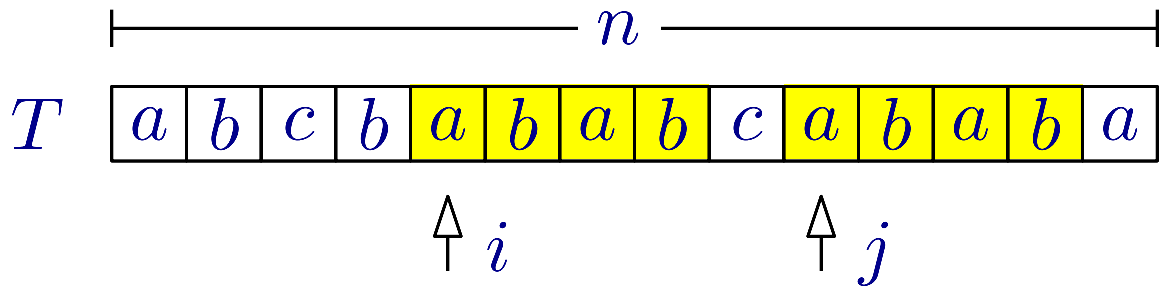

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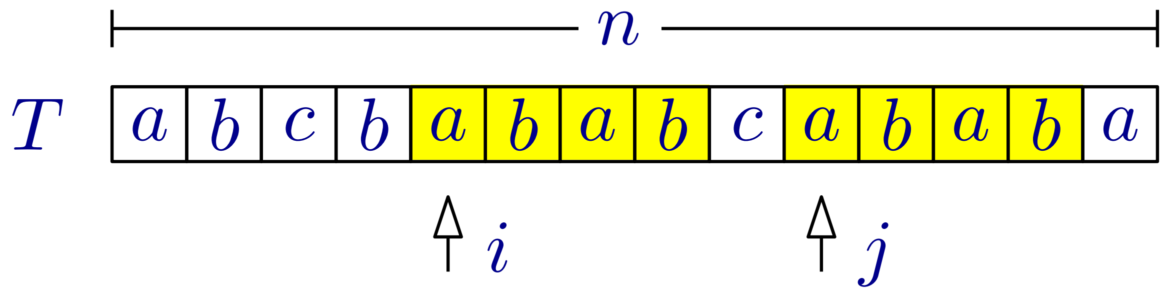

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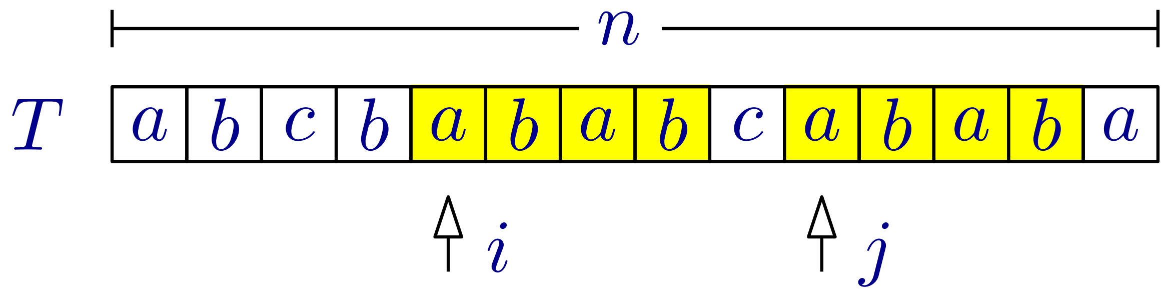

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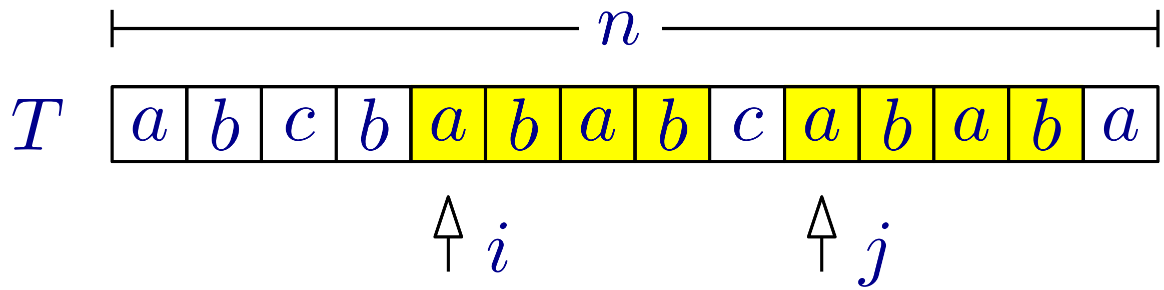

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- LCP data structures are typically based
  on the suffix array or suffix tree.
- We do the opposite - we use *batched* LCP queries
  to construct the sparse suffix array
- These LCP queries will be answered using Karp-Rabin fingerprints
  to ensure that the space remains small

Karp-Rabin fingerprints of strings

S

a	b	a	c	c	b	a	b	c	b

$$\phi(S) = \sum_{k=0}^{|S|-1} S[k]r^k \bmod p$$

Here $p = \Theta(n^4)$ is a prime and $1 \leq r < p$ is a random integer

with high probability, $S_1 = S_2$ iff $\phi(S_1) = \phi(S_2)$

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Given $\phi(S[0, \ell])$ and $\phi(S[0, r])$ we can compute $\phi(S[\ell + 1, r])$ in $O(1)$ time

Simple, Monte-Carlo batched LCP queries

Input : a string, T of length n and b pairs, (i, j)

Output : for each pair (i, j) output the largest ℓ s.t.

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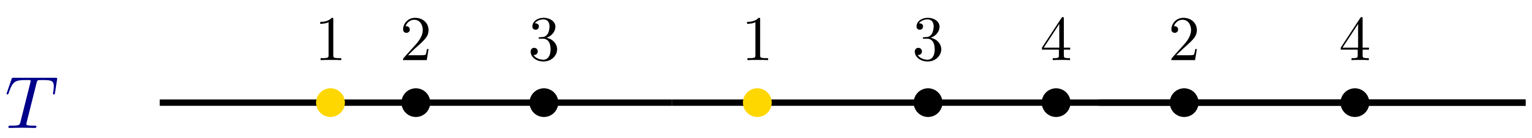

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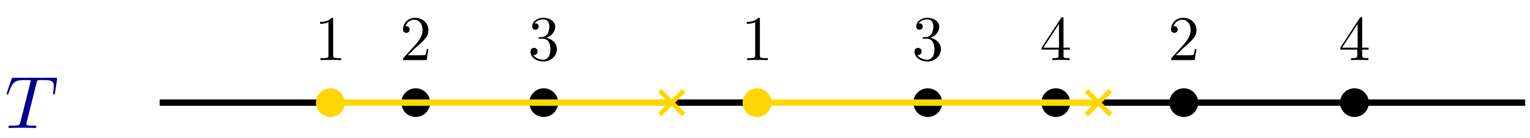

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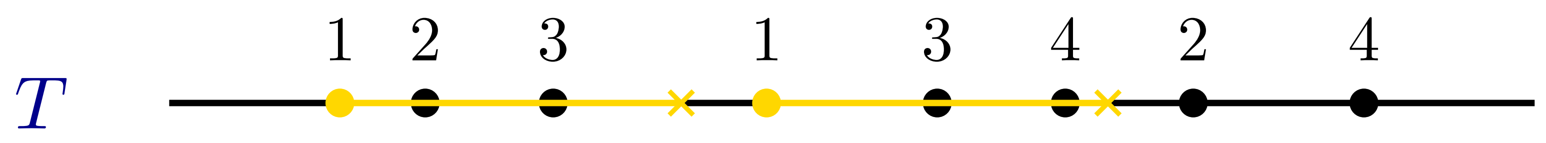

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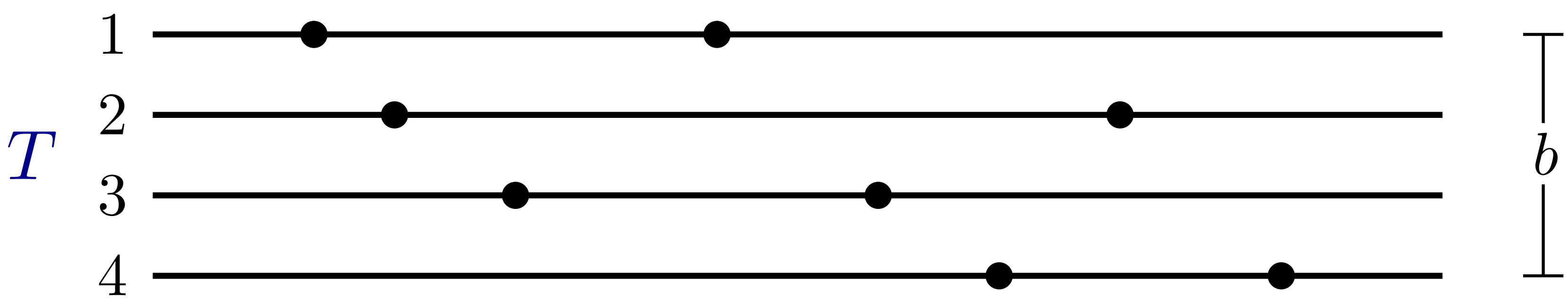

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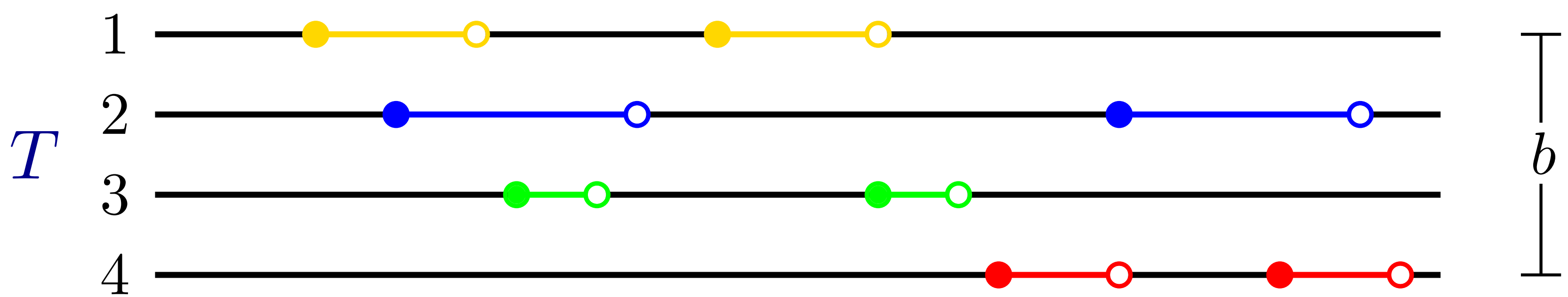

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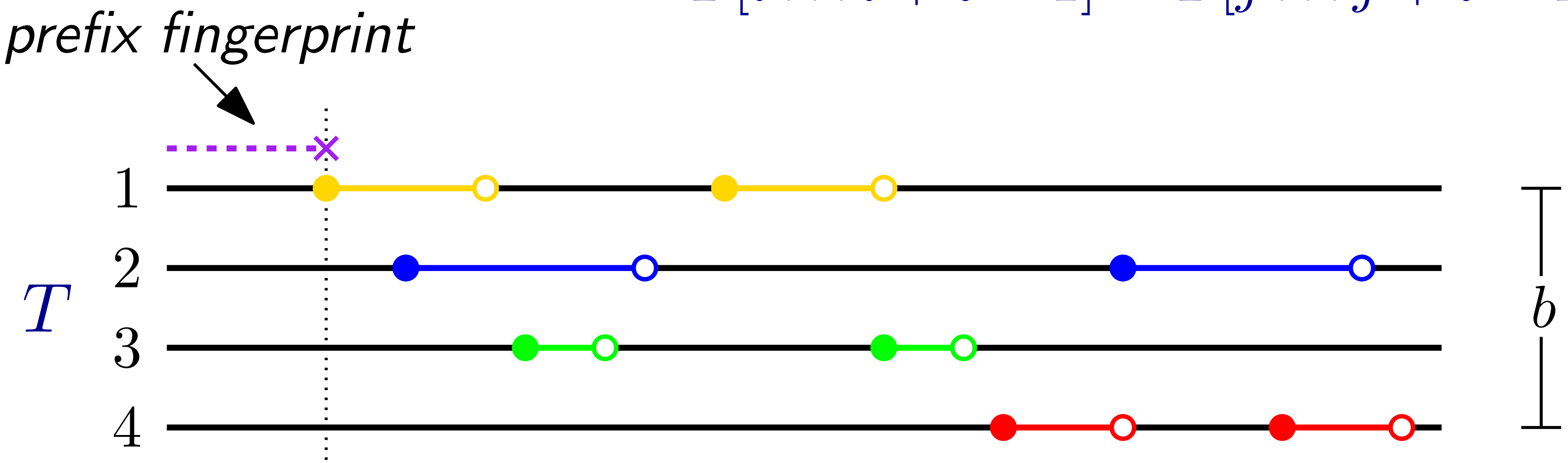

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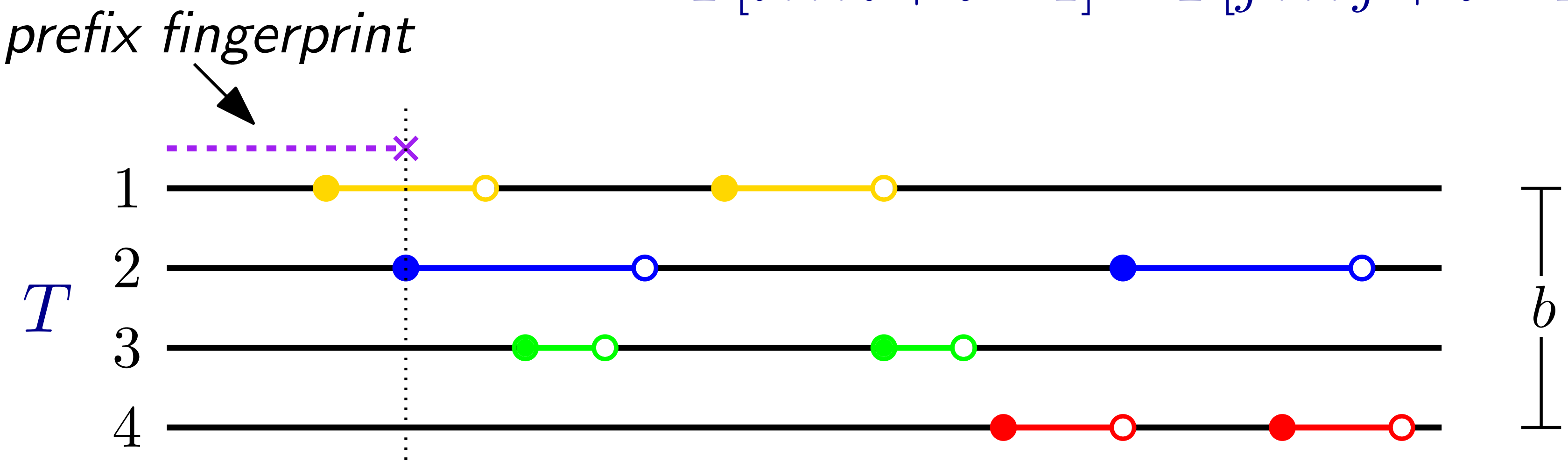

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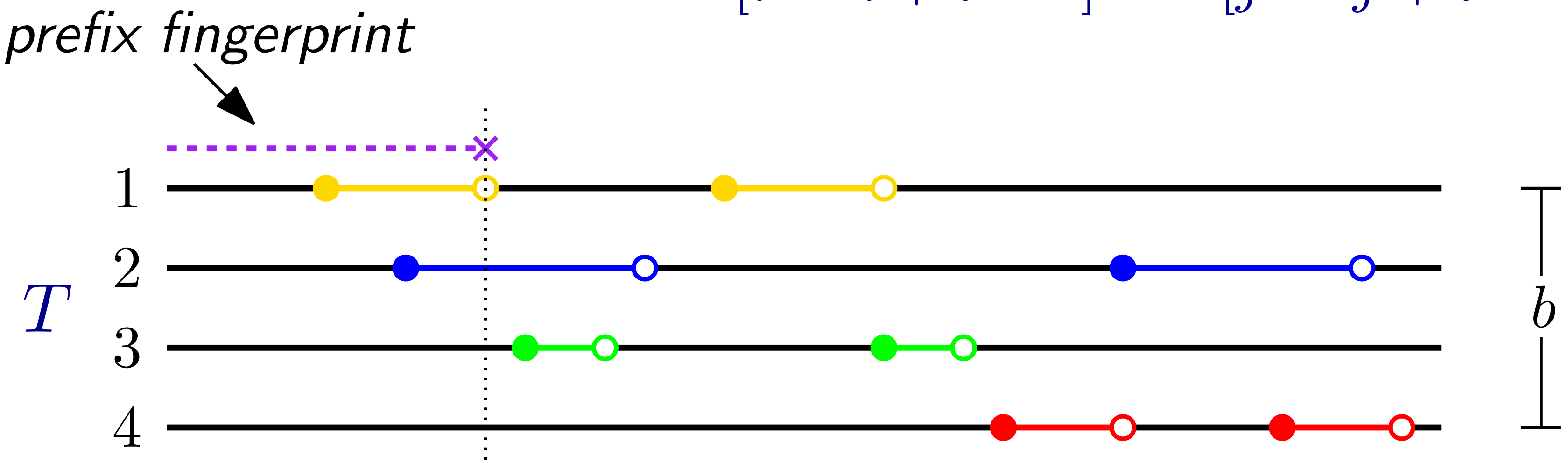

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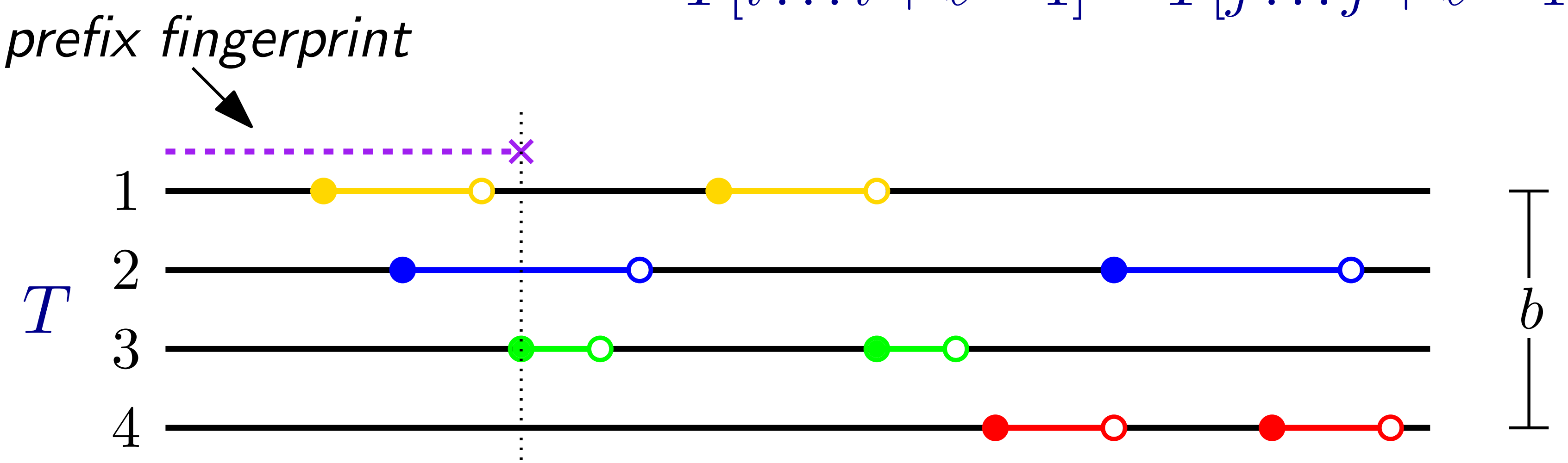

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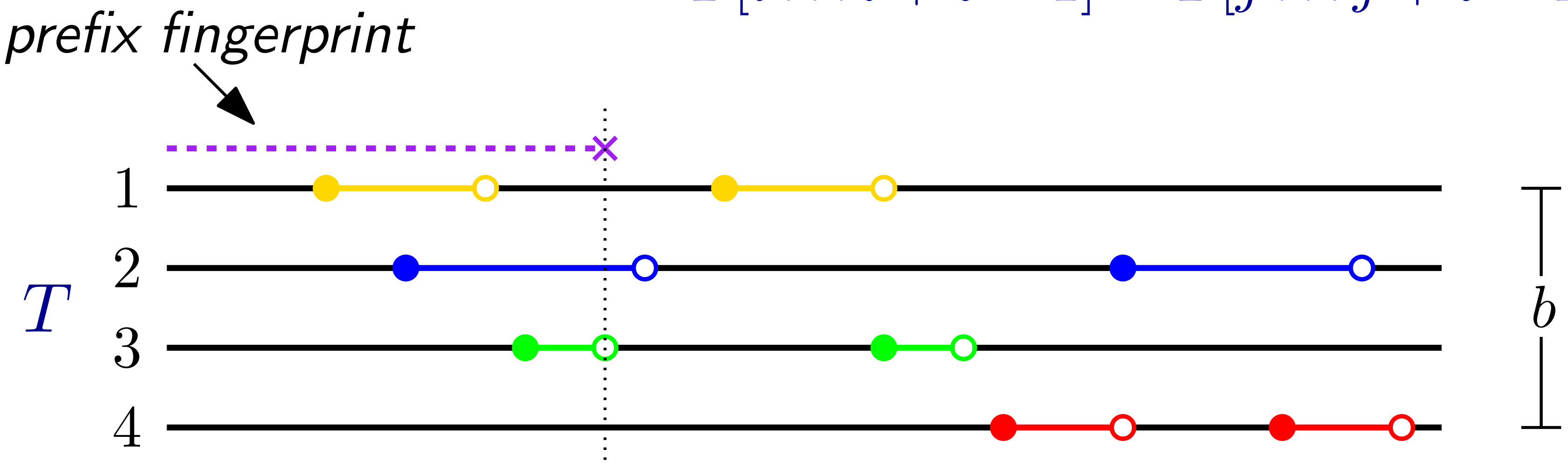

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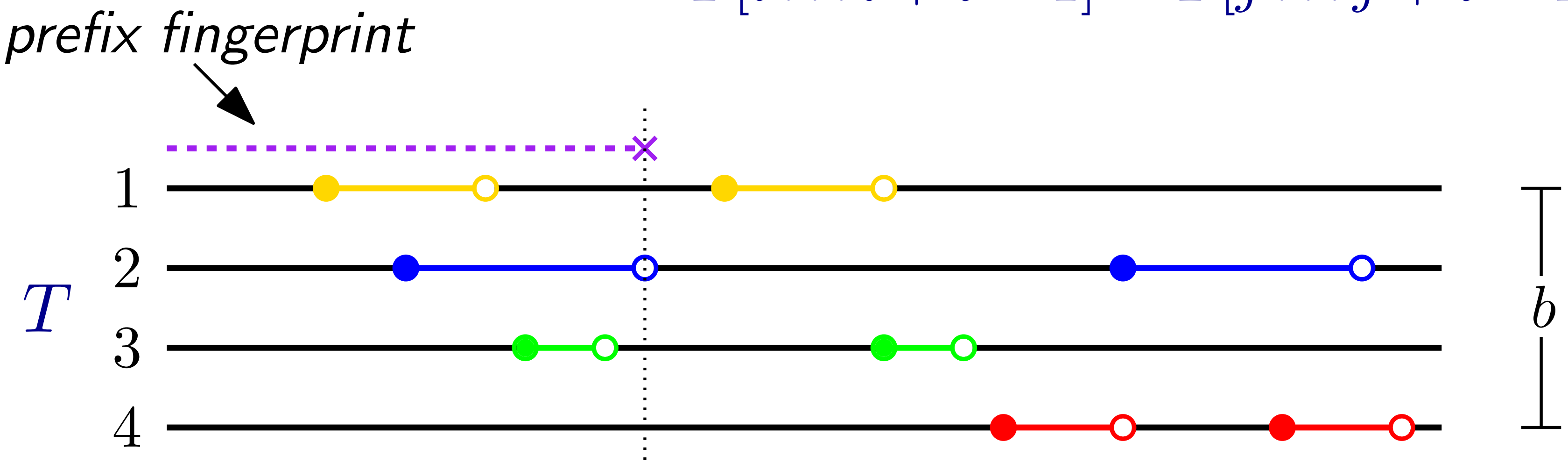

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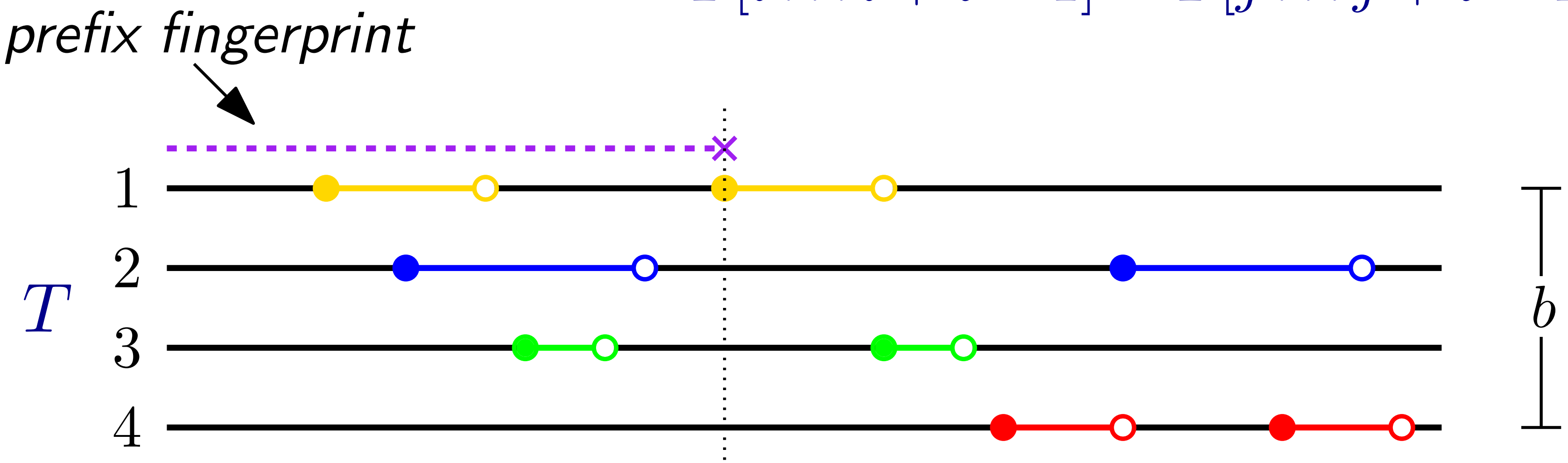

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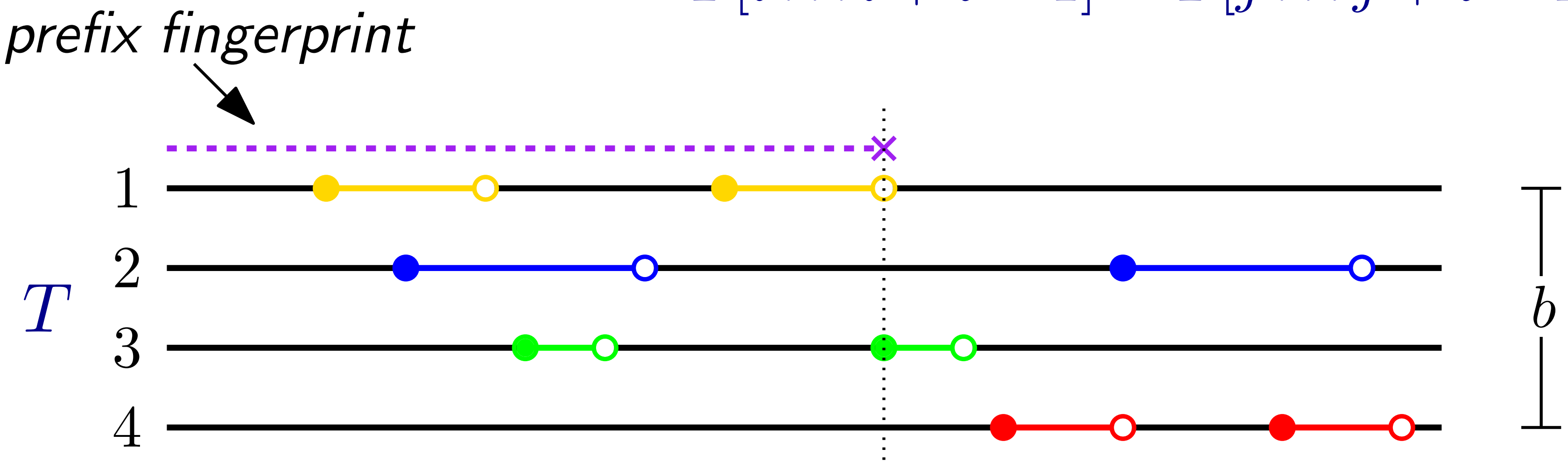

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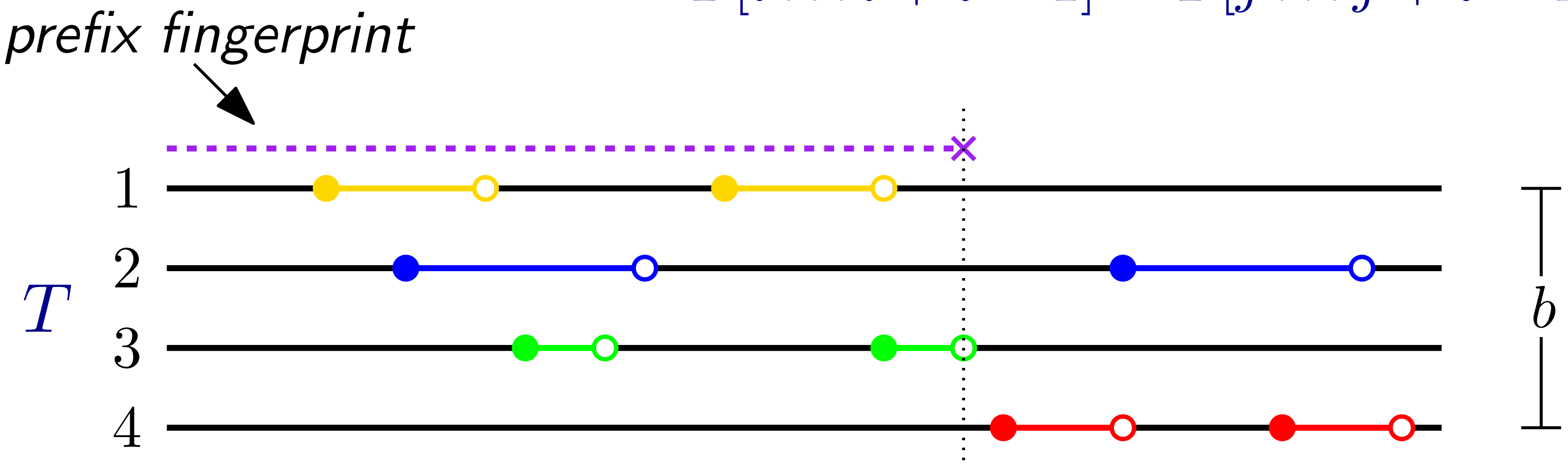

- We find the largest ℓ for each pair by binary search (in parallel) comparisons are performed using fingerprints

Simple, Monte-Carlo batched LCP queries

Input : a string, T of length n and b pairs, (i, j)

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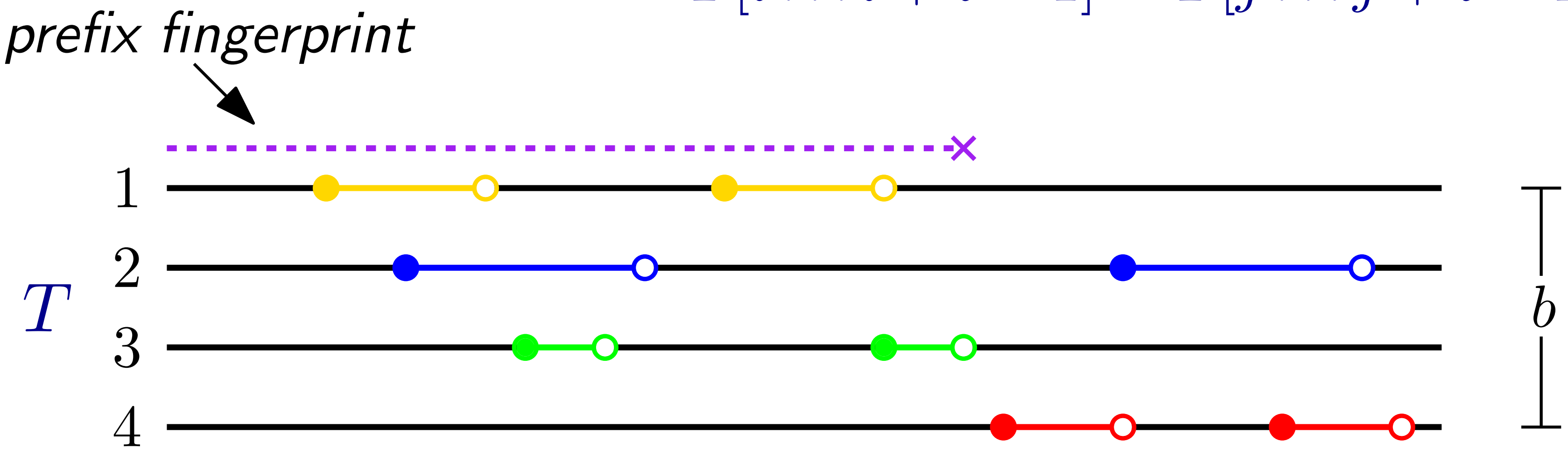

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- In each pass we store (at most) $4b$ prefix fingerprints

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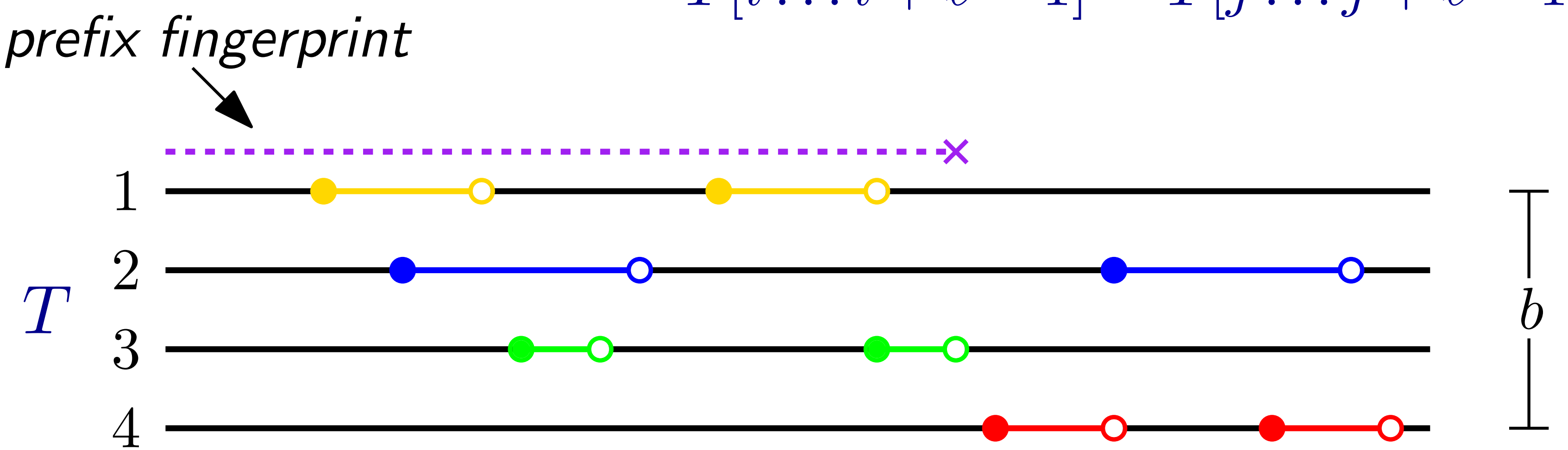

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this takes $O(n \log b)$ time, $O(b)$ space and is correct whp.

Building the sparse suffix array using batched LCPs

T	b	a	n	a	n	a	s

Building the sparse suffix array using batched LCPs

T

b	a	n	a	n	a	s

1	b	a	n	a	n	a	s
2	a	n	a	n	a	s	
3	n	a	n	a	s		
4	a	n	a	s			
5	n	a	s				
6	a	s					
7	s						

Building the sparse suffix array using batched LCPs

T	b	a	n	a	n	a	s

1	b	a	n	a	n	a	s
2	a	n	a	n	a	s	
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Building the sparse suffix array using batched LCPs

T	b	a	n	a	n	a	s

The LCP of two suffixes gives us their order

1	b	a	n	a	n	a	s
2	a	n	a	n	a	s	
3	n	a	n	a	s		
4	a	n	a	s			
5	n	a	s				
6	a	s					
7	s						

Building the sparse suffix array using batched LCPs

T							
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The LCP of two suffixes gives us their order

1	b	a	n	a	n	a	s
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4	a	n	a	s			
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6	a	s					
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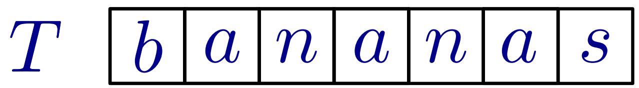
Building the sparse suffix array using batched LCPs



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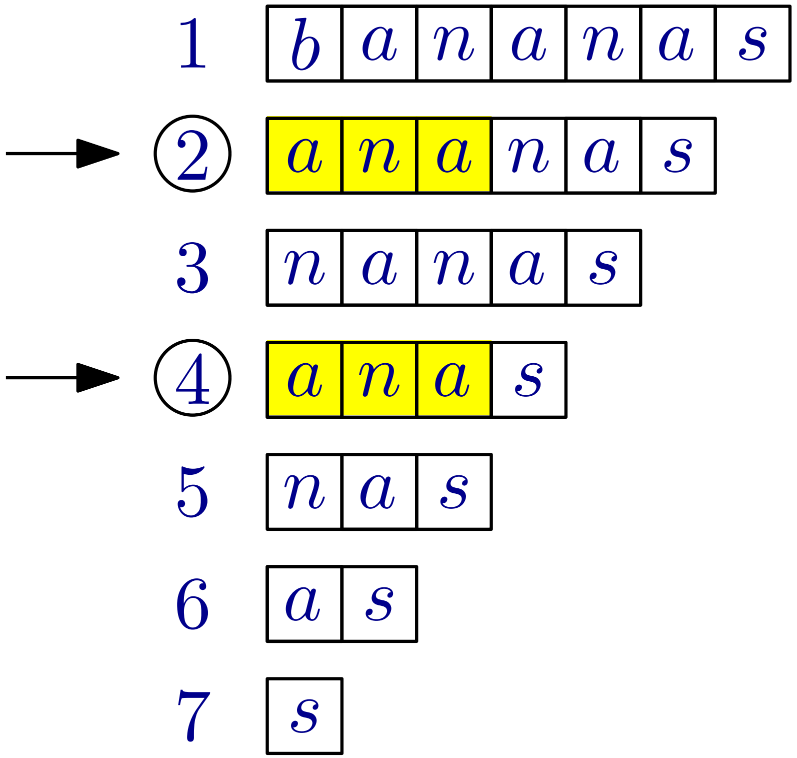
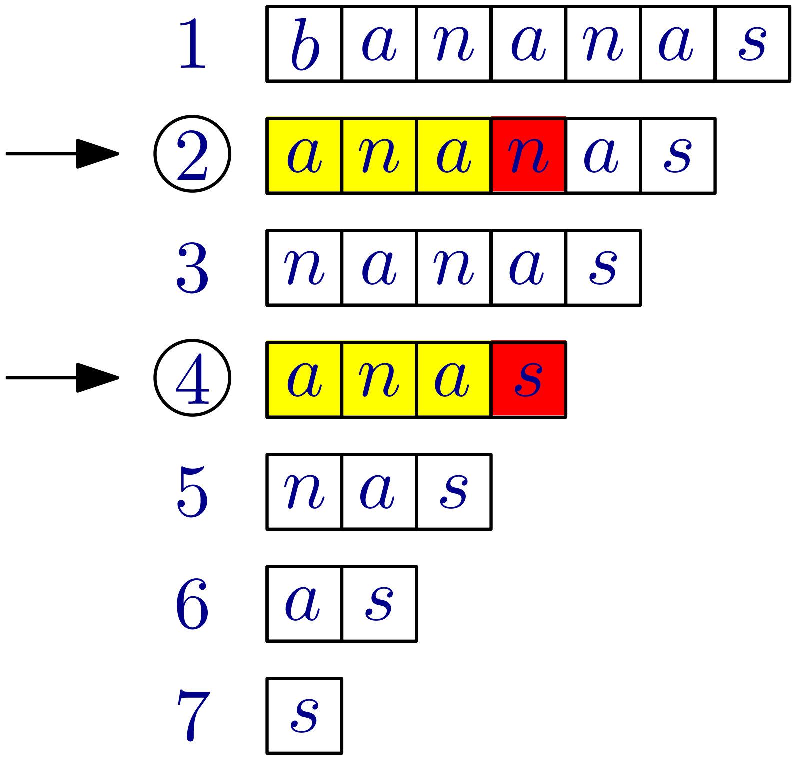


Building the sparse suffix array using batched LCPs

T | b | a | n | a | n | a | s |

The LCP of two suffixes gives us their order

②<④ because $\boxed{n} < \boxed{s}$



Building the sparse suffix array using batched LCPs

T $\boxed{b}\boxed{a}\boxed{n}\boxed{a}\boxed{n}\boxed{a}\boxed{s}$

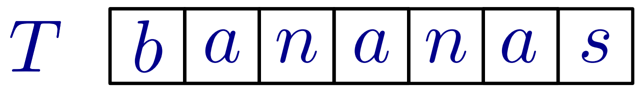
1	$\boxed{b}\boxed{a}\boxed{n}\boxed{a}\boxed{n}\boxed{a}\boxed{s}$
→ 2	$\boxed{a}\boxed{n}\boxed{a}\boxed{n}\boxed{a}\boxed{s}$
3	$\boxed{n}\boxed{a}\boxed{n}\boxed{a}\boxed{s}$
→ 4	$\boxed{a}\boxed{n}\boxed{a}\boxed{s}$
5	$\boxed{n}\boxed{a}\boxed{s}$
6	$\boxed{a}\boxed{s}$
7	$\boxed{s}$

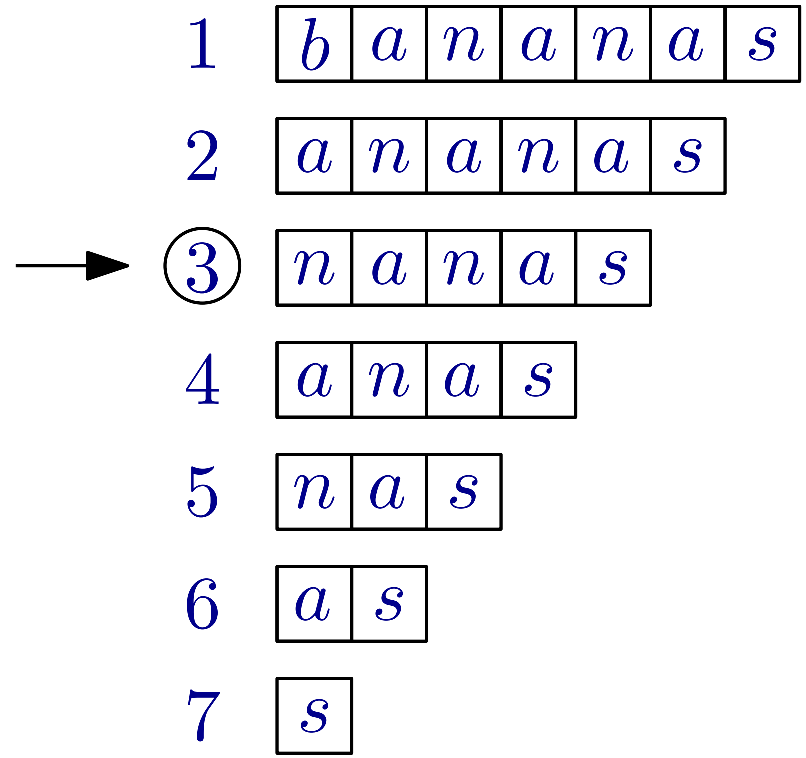
The LCP of two suffixes gives us their order

$2 < 4$ because $\boxed{n} < \boxed{s}$

- We perform randomised quicksort on the b suffixes using batched LCPs for suffix comparisons

Building the sparse suffix array using batched LCPs

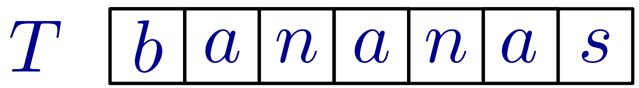




The LCP of two suffixes gives us their order

- We perform randomised quicksort on the b suffixes using batched LCPs for suffix comparisons
- Pick a random pivot and compare each other suffix to it
  - This partitions the suffixes in $O(n \log b)$ time and $O(b)$ space

Building the sparse suffix array using batched LCPs



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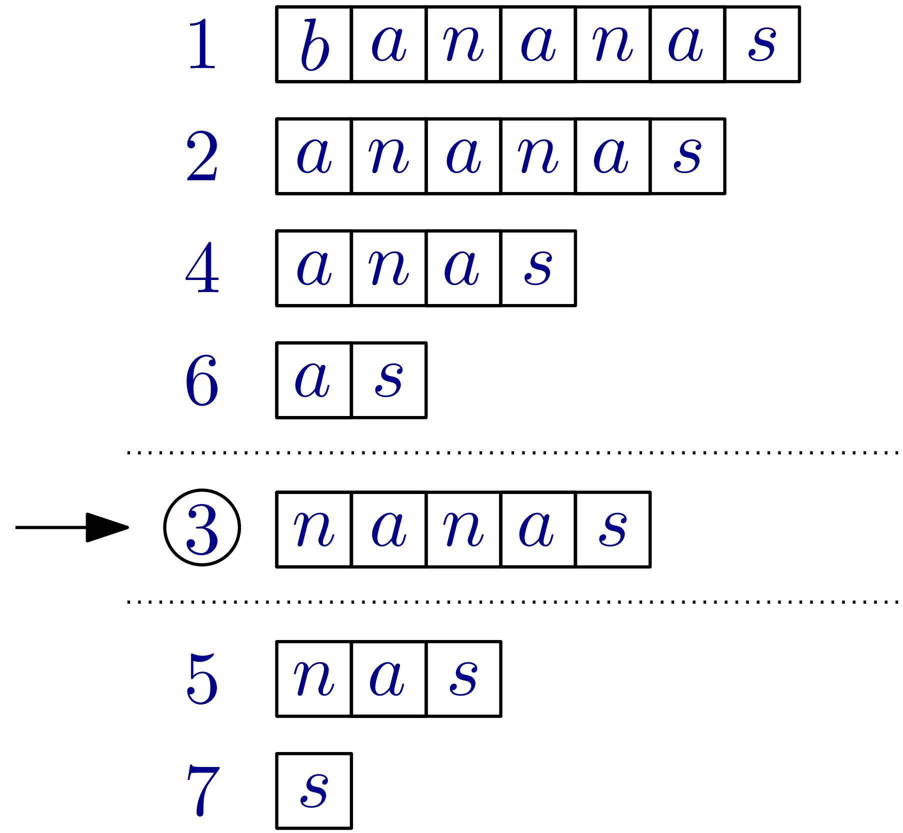

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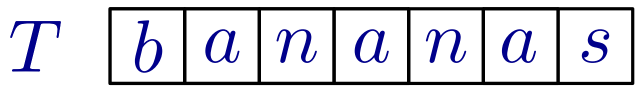

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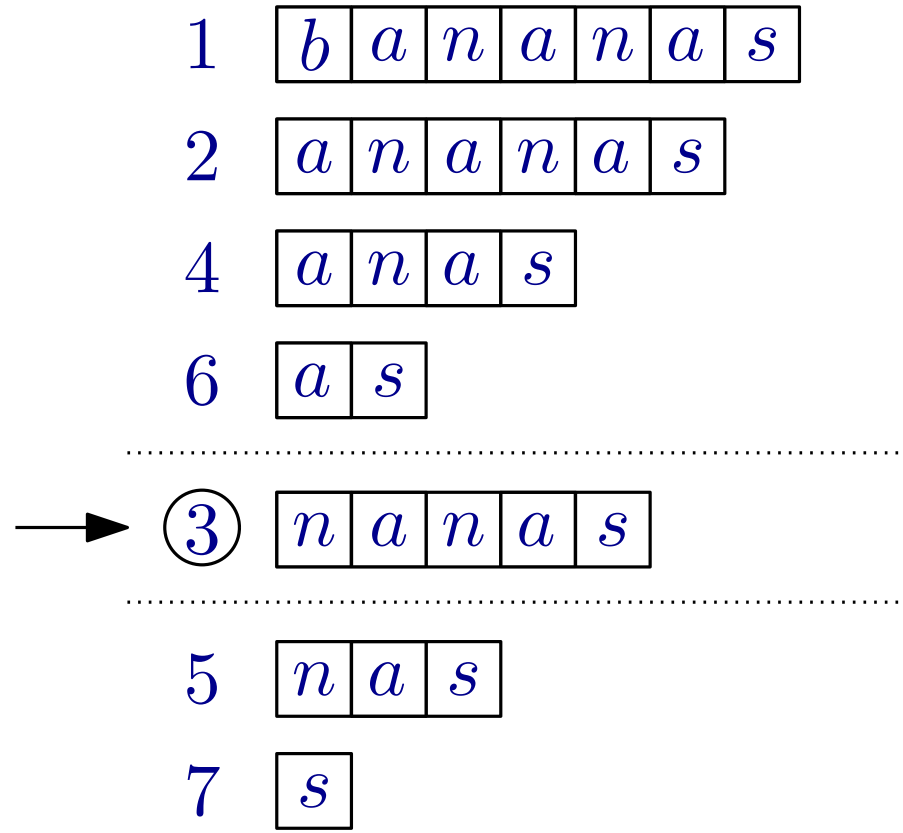

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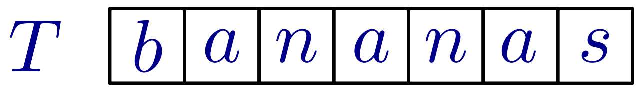
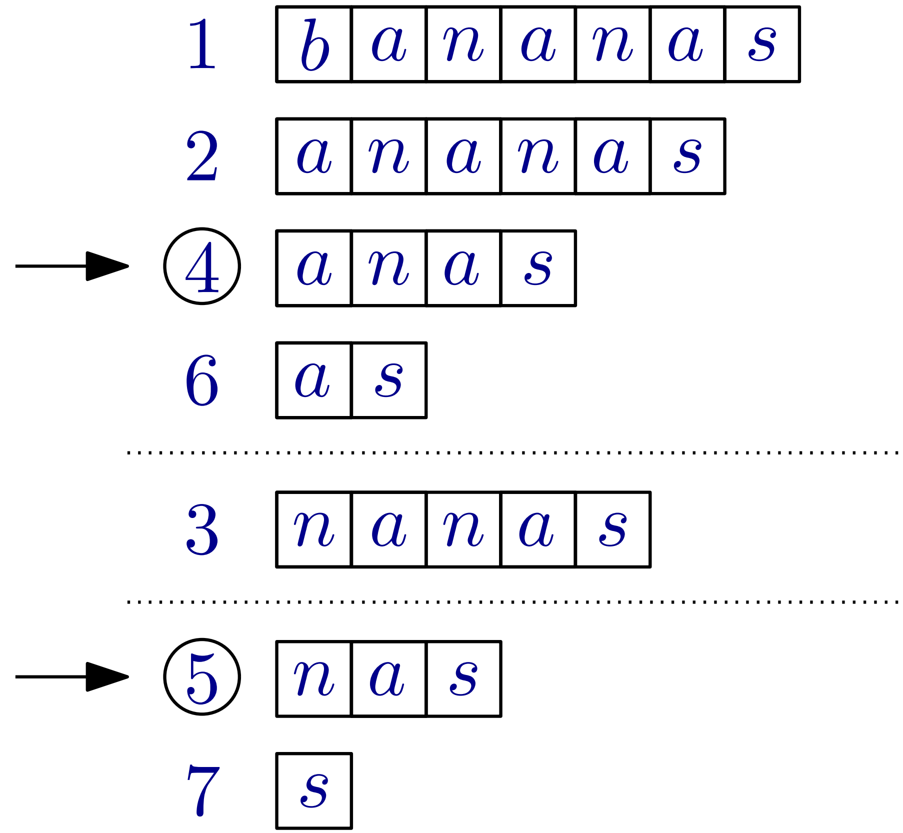
Building the sparse suffix array using batched LCPs





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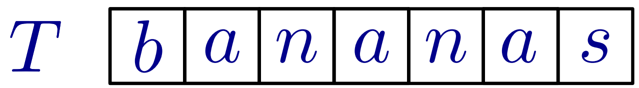

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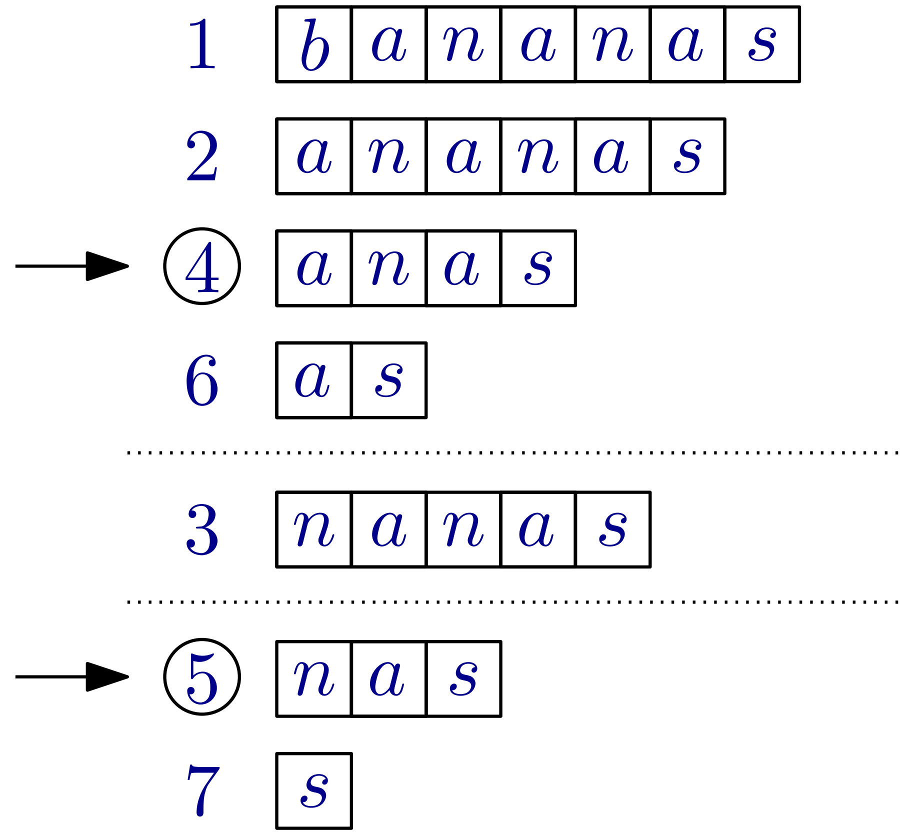
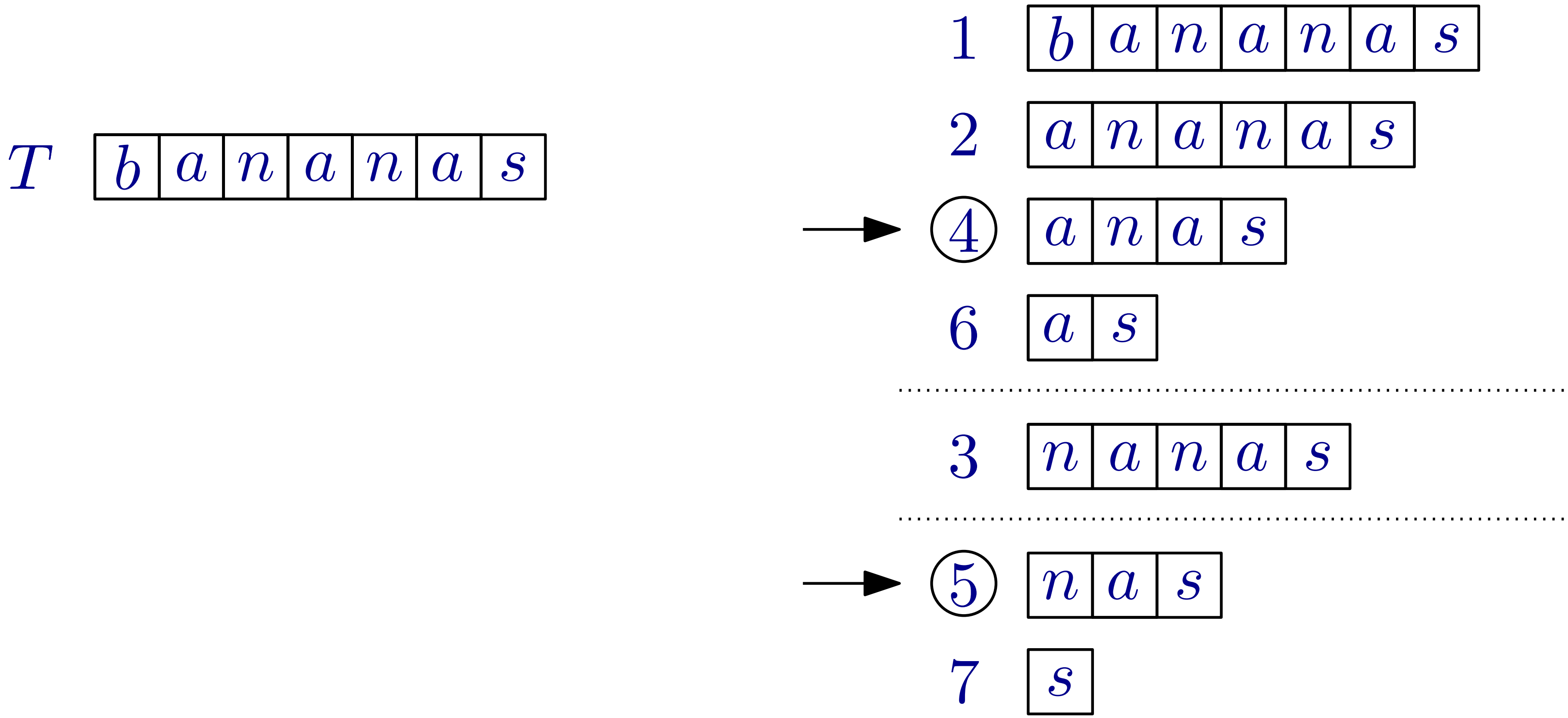


- We perform randomised quicksort on the b suffixes using batched LCPs for suffix comparisons
- The depth of the recursion is $O(\log b)$ whp. so...

  The total time is $O(n \log^2 b)$ and the space is $O(b)$

Building the sparse suffix array using batched LCPs



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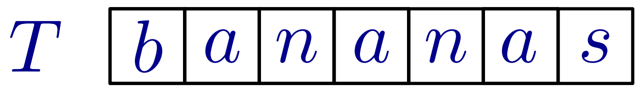

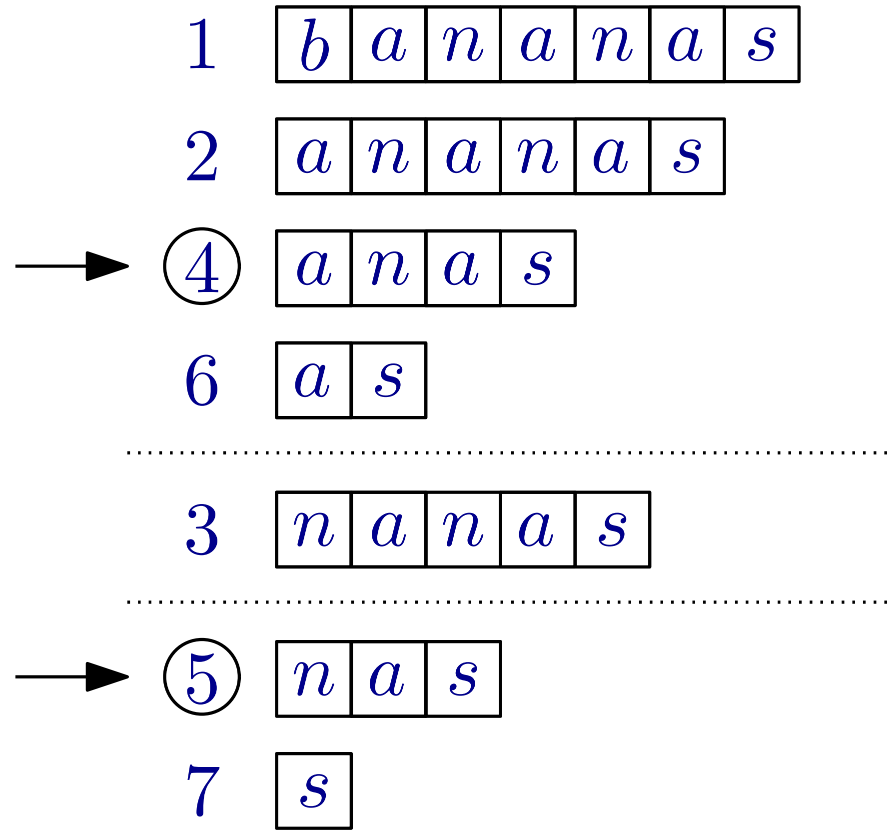

This algorithm is Monte-Carlo *and* Las-Vegas. It can be made Monte-Carlo only by aborting the quicksort early

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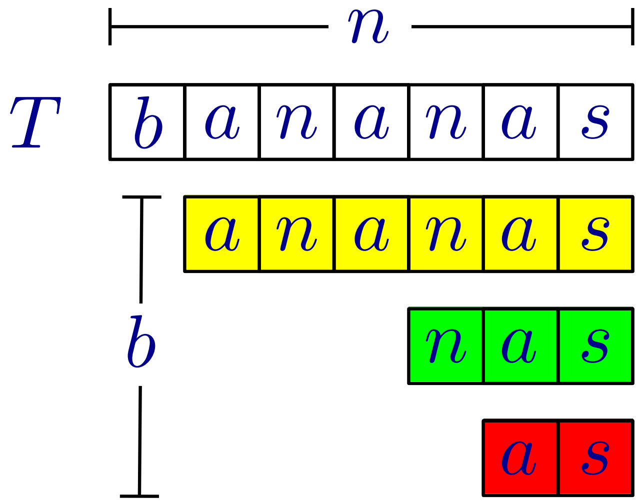
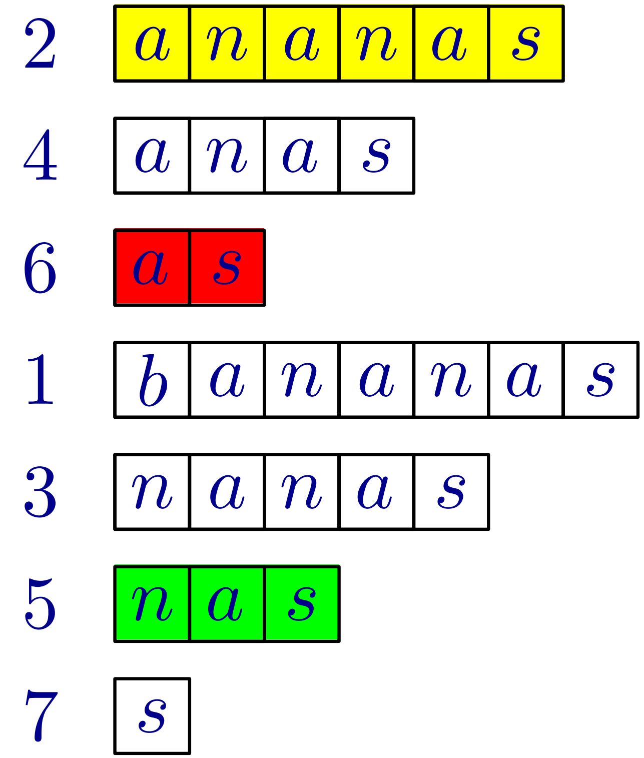
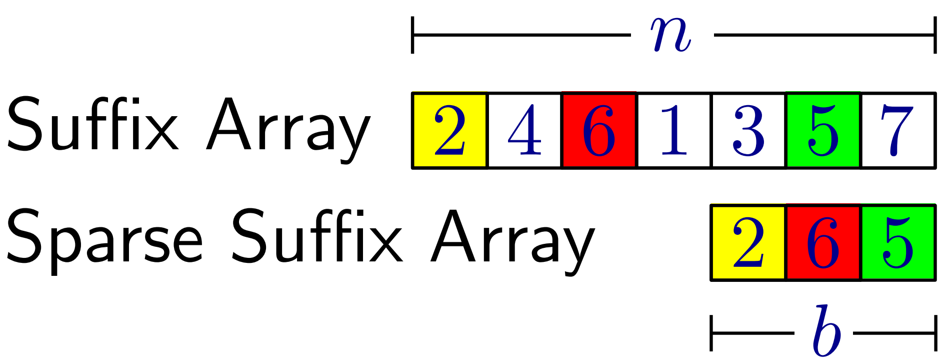
The total time is $O(n \log^2 b)$ and the space is $O(b)$

The sparse suffix array (SSA)







- $O(n \log^2 b)$ time (Monte-Carlo)
- $O((n + b^2) \log^2 b)$ time with high probability (Las-Vegas)
- both in $O(b)$ space

Verifying the sparse suffix array

T	b	a	n	a	n	a	s
Suffix Array	2	4	6	1	3	5	7

n

How can we tell if this suffix array is correct?

Verifying the sparse suffix array

T	b	a	n	a	n	a	s
	$\vdash$			n			$\dashv$
Suffix Array	2	4	6	1	3	5	7

How can we tell if this suffix array is correct?

Check that $\boxed{2} < \boxed{4}\,,\ \boxed{4} < \boxed{6}\,,\ \boxed{6} < \boxed{1}\,,\ \boxed{1} < \boxed{3}\ \ldots$

Verifying the sparse suffix array

T	b	a	n	a	n	a	s
	$\longleftarrow$			n			$\longrightarrow$
Suffix Array	2	4	6	1	3	5	7

How can we tell if this suffix array is correct?

Check that $\boxed{2} < \boxed{4}$, $\boxed{4} < \boxed{6}$, $\boxed{6} < \boxed{1}$, $\boxed{1} < \boxed{3}$ $\ldots$

Verifying the sparse suffix array

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We could check $\boxed{2} < \boxed{4}$ using an LCP query if we verified it

Verifying the sparse suffix array

$$T \quad \boxed{b}\boxed{a}\boxed{n}\boxed{a}\boxed{n}\boxed{a}\boxed{s}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{n}$$

Suffix Array $\boxed{2}\boxed{4}\boxed{6}\boxed{1}\boxed{3}\boxed{5}\boxed{7}$

How can we tell if this suffix array is correct?

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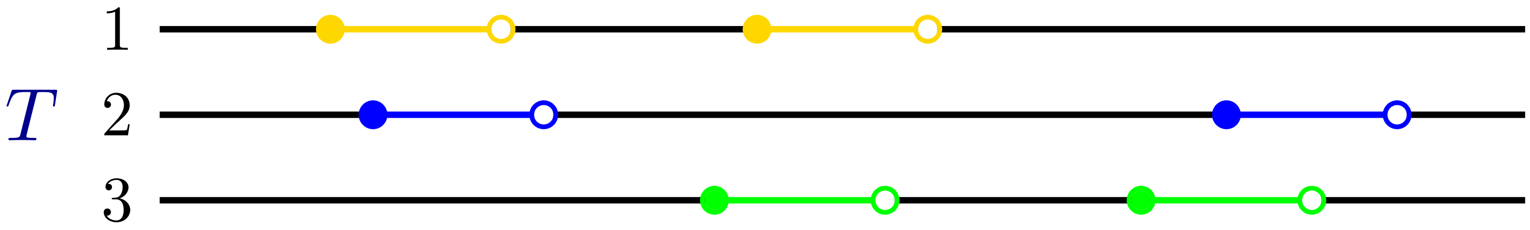
A first example



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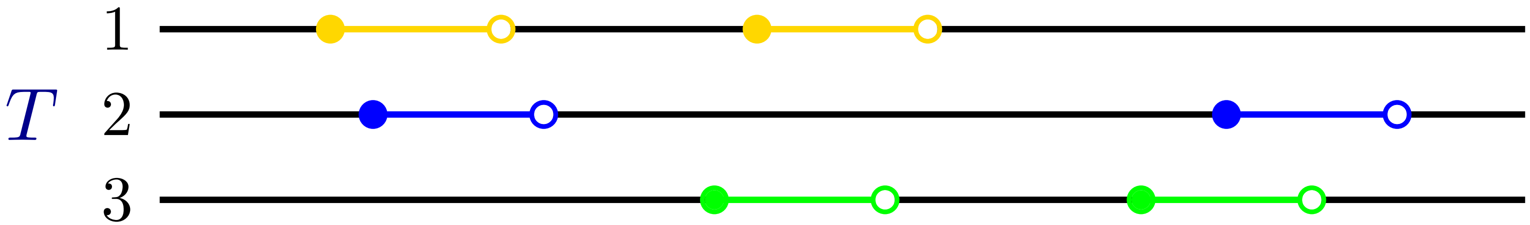

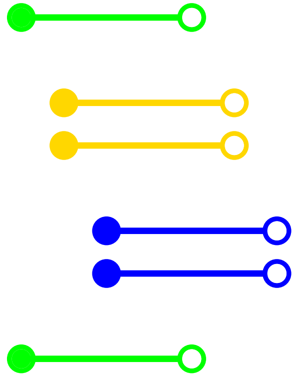

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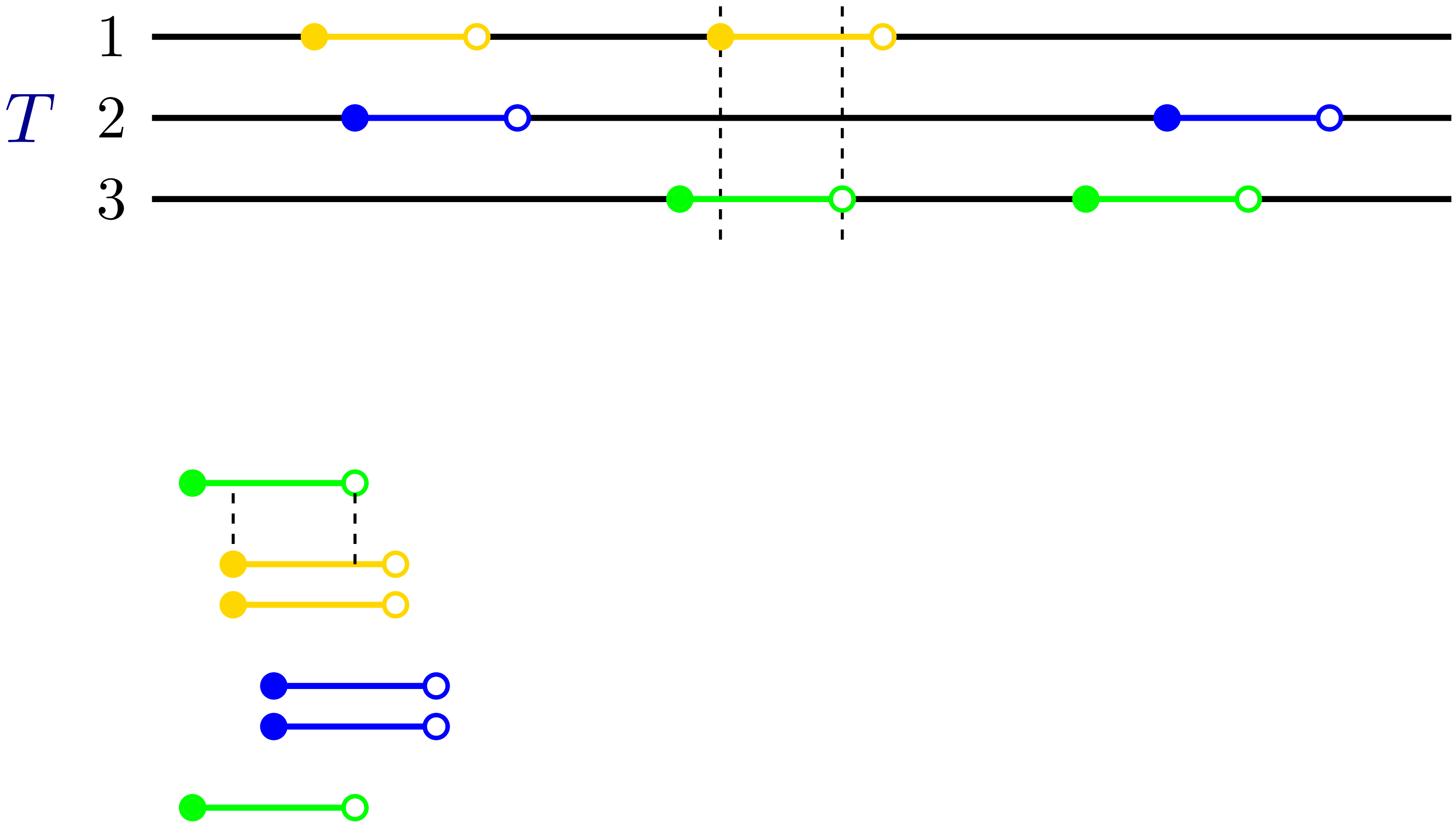

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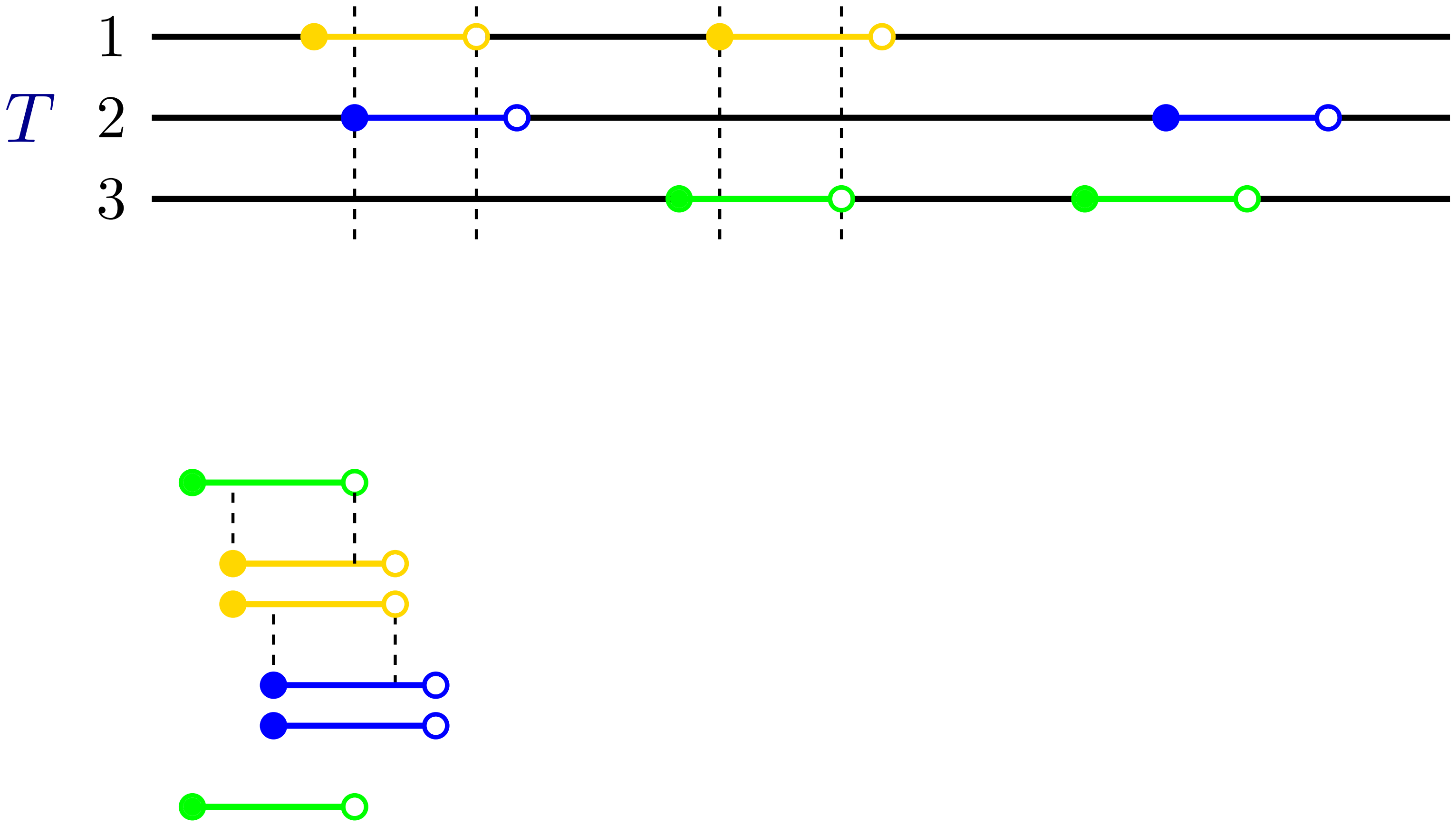

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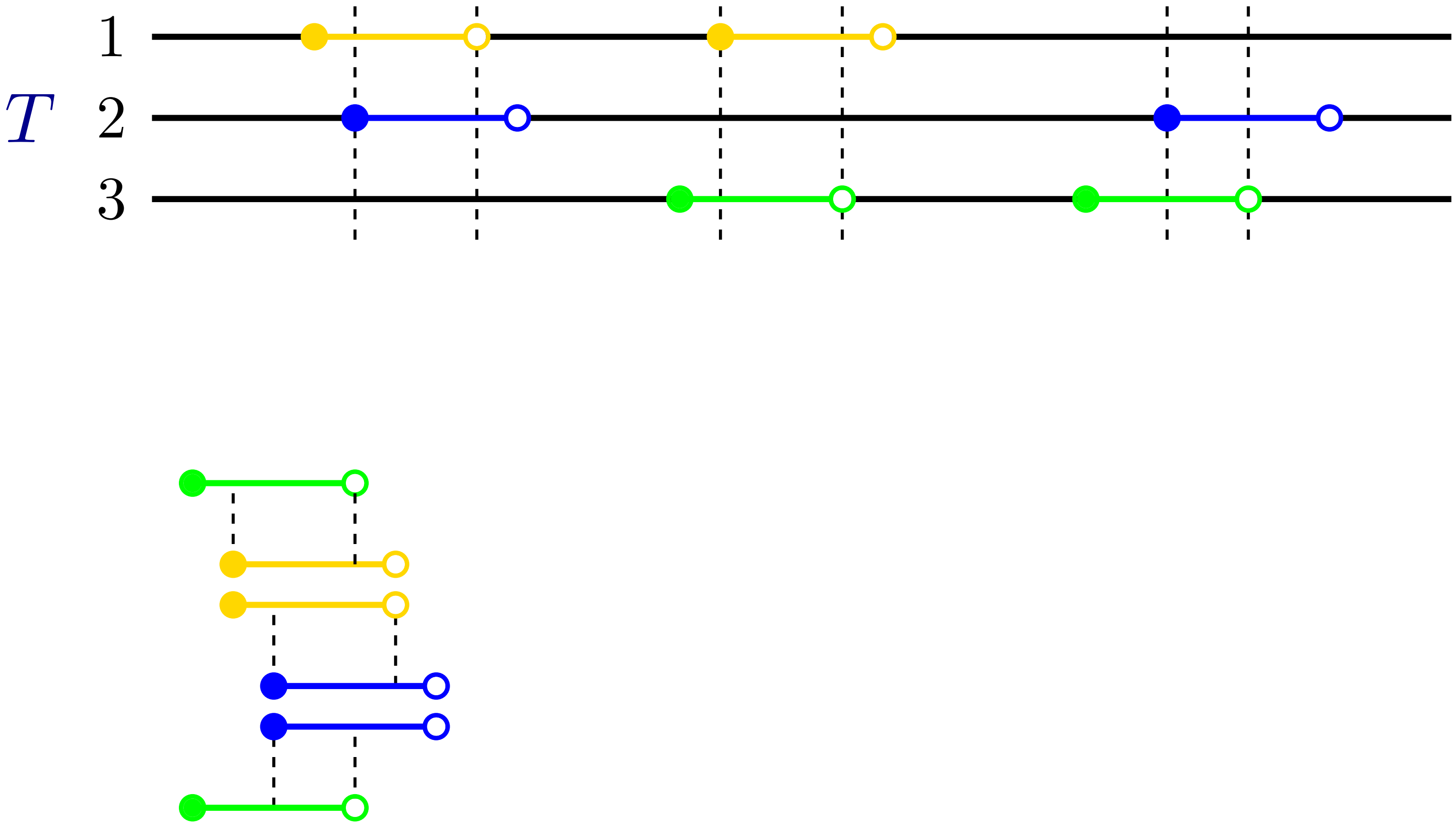

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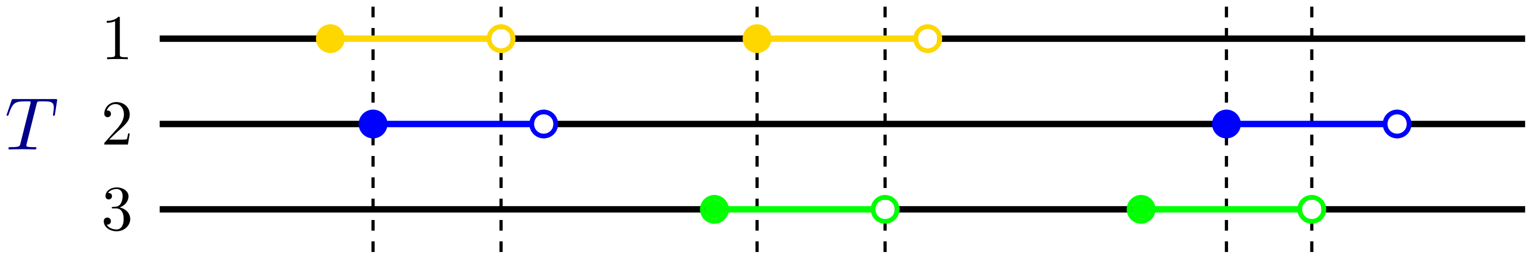
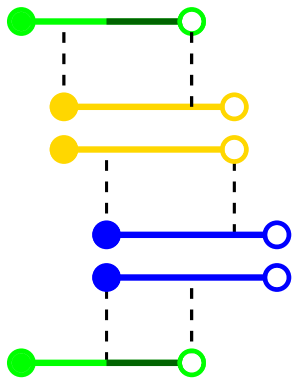
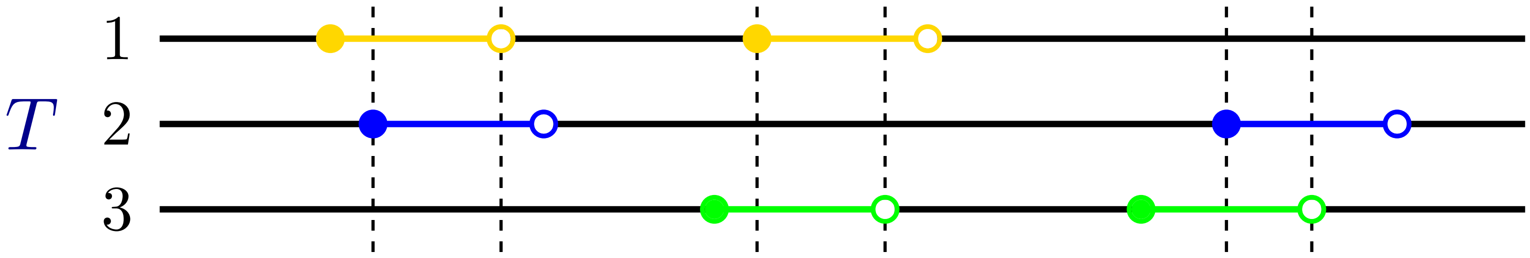
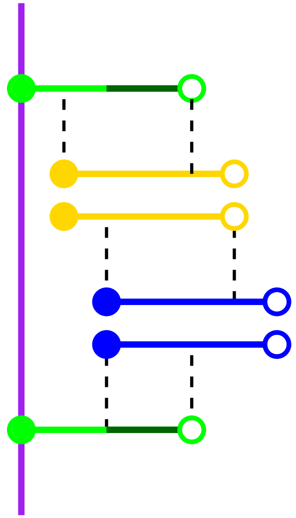




If yellow (1) and blue (2) match then the right half of green (3) matches

A first example

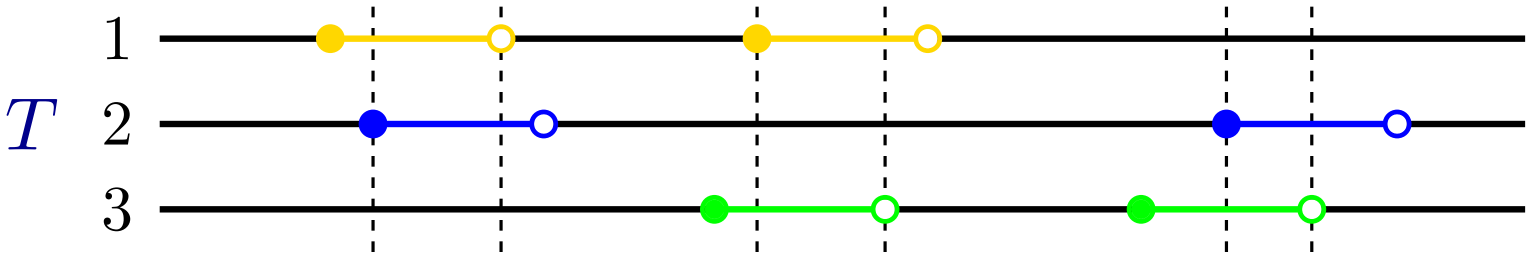


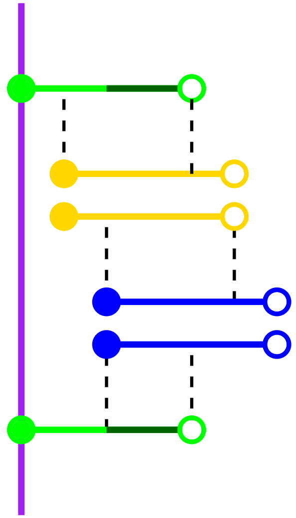


If yellow (1) and blue (2) match then the right half of green (3) matches

This is a *lock-stepped* cycle

A first example





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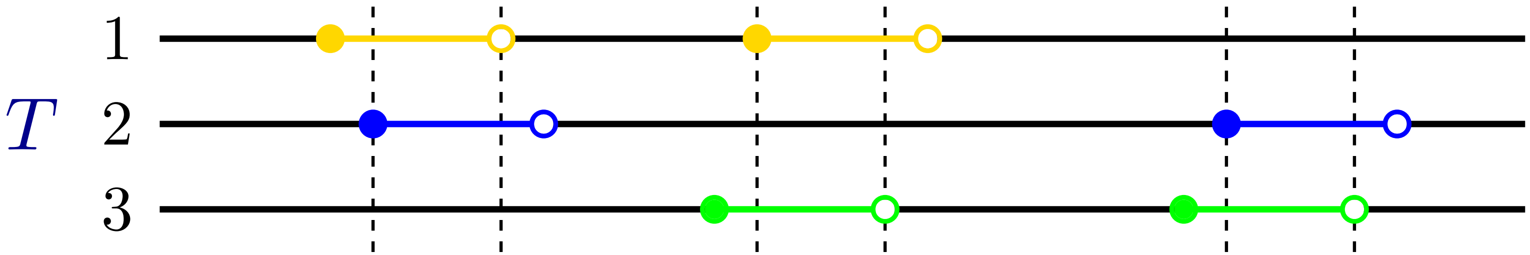

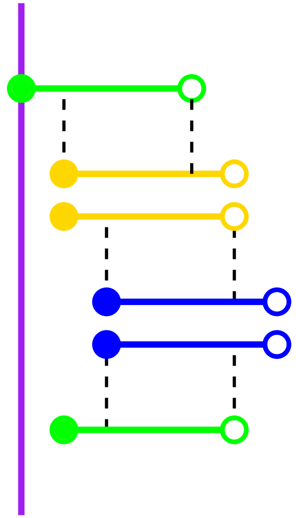

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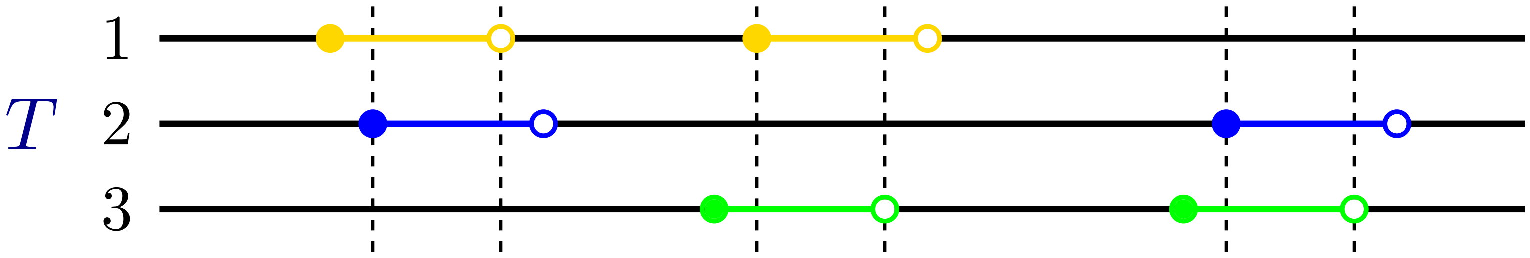
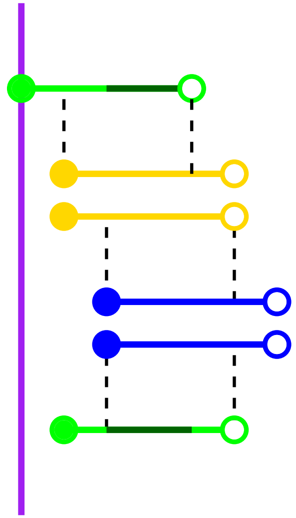
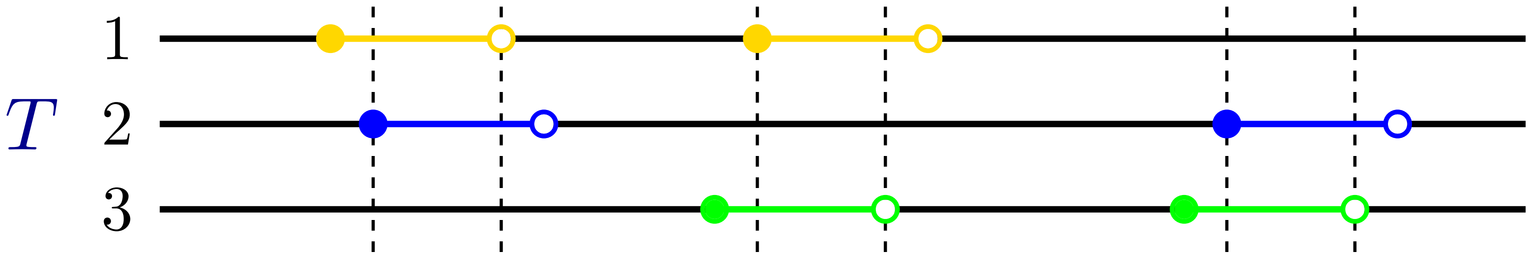
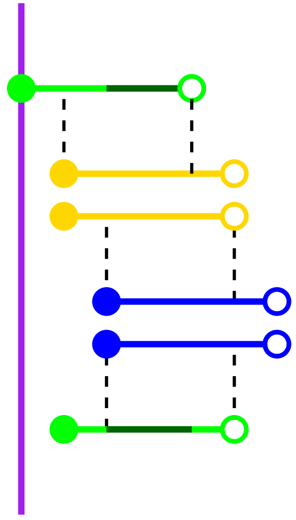




If yellow (1),blue (2) and green (3) match then $\frac{3}{4}$ of green (3) is periodic

A second example

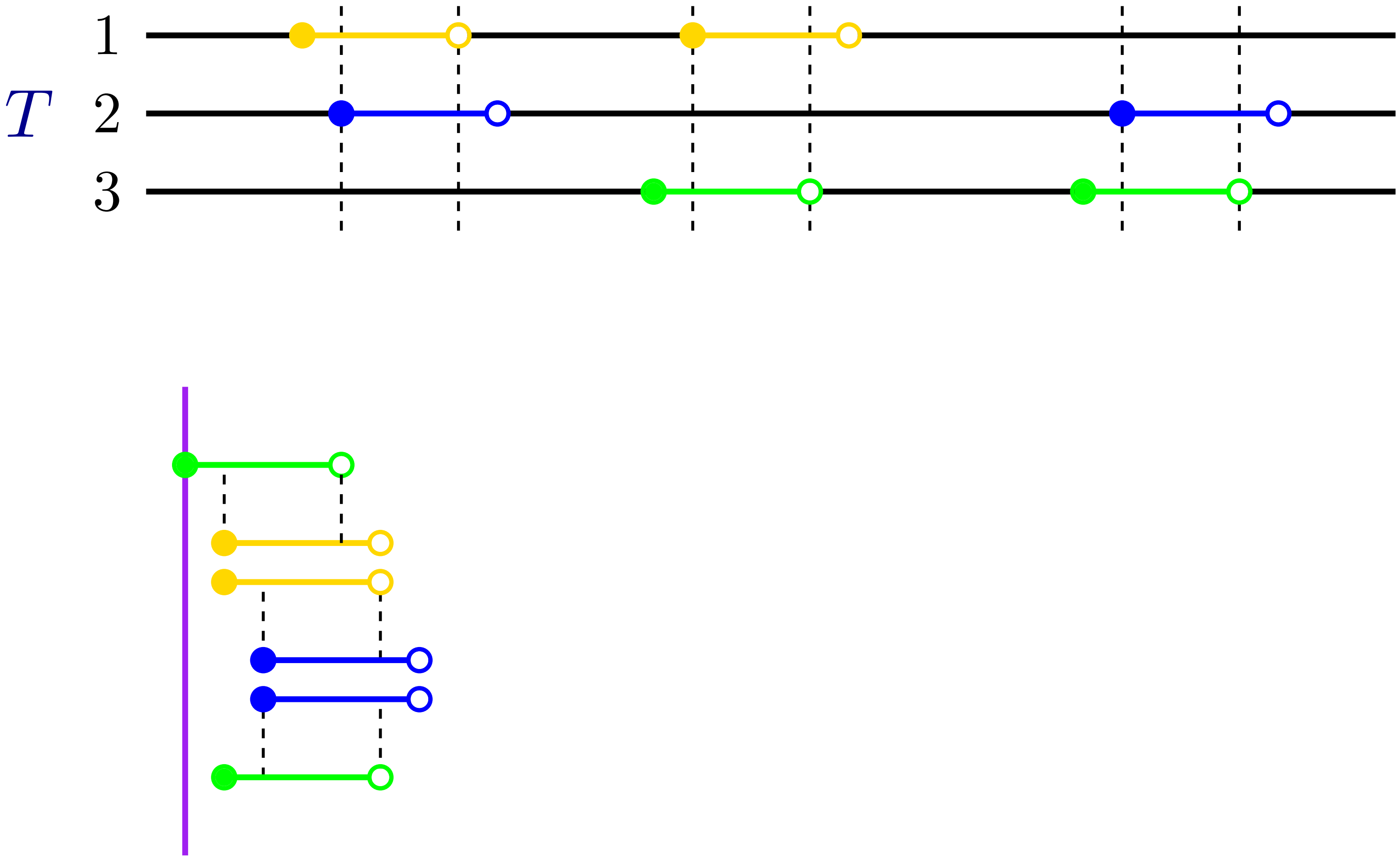




If yellow (1), blue (2) and green (3) match then $\frac{3}{4}$ of green (3) is periodic

This is an *unlocked* cycle

A second example



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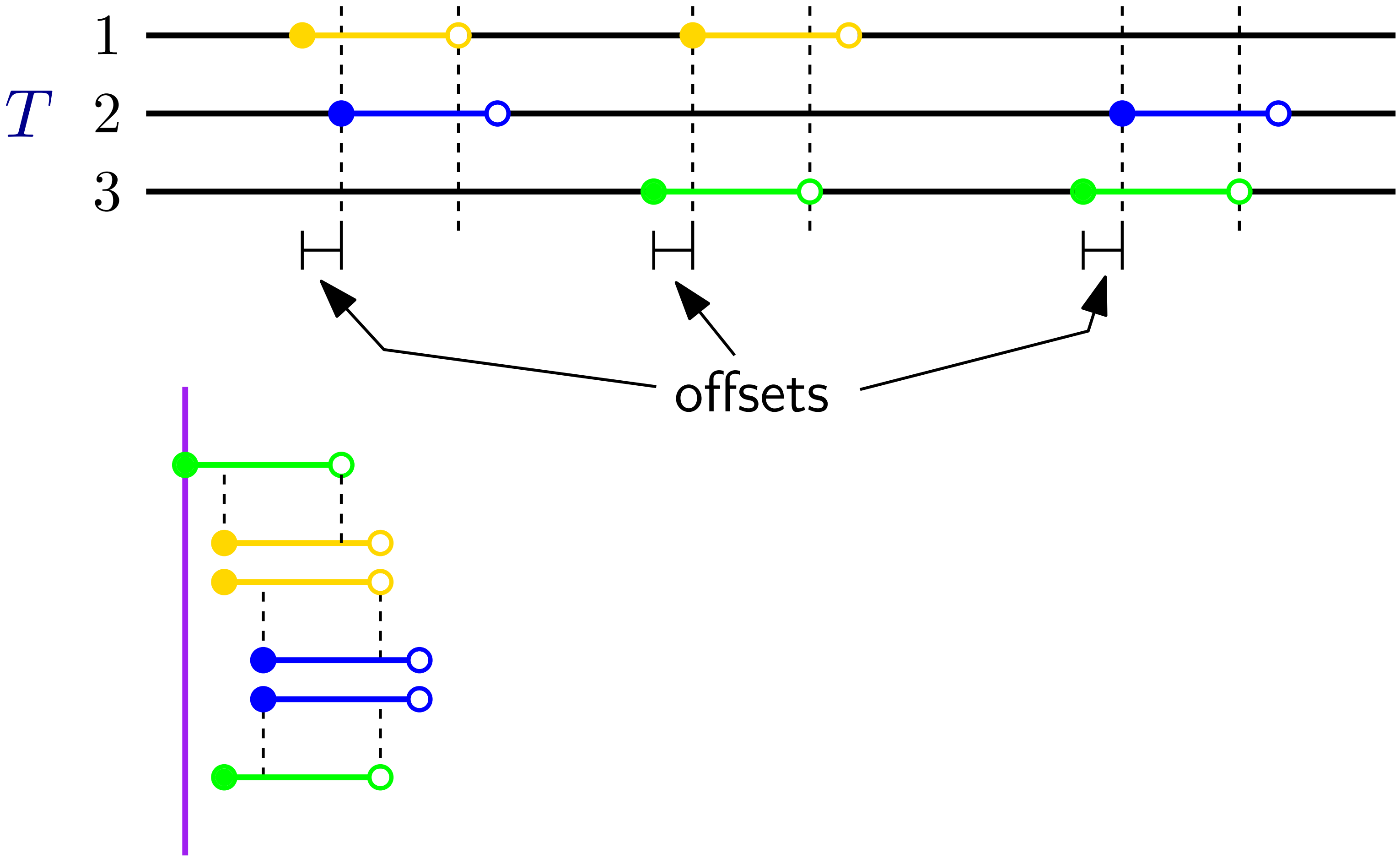
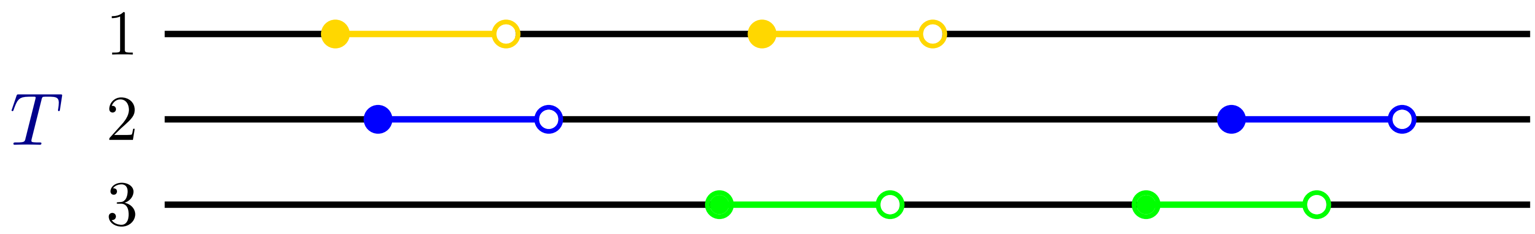


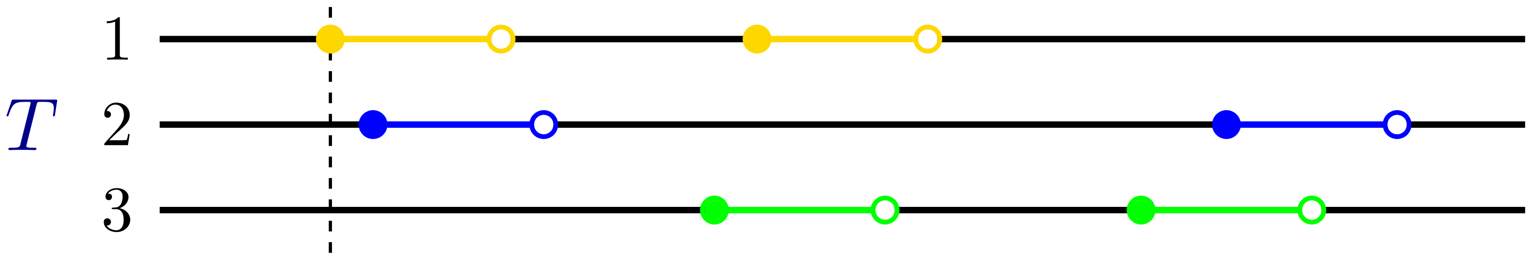
These tricks only work when the offsets are *small*

The overall idea



- We build a graph which encodes the structure of the queries

The overall idea



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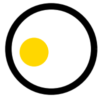

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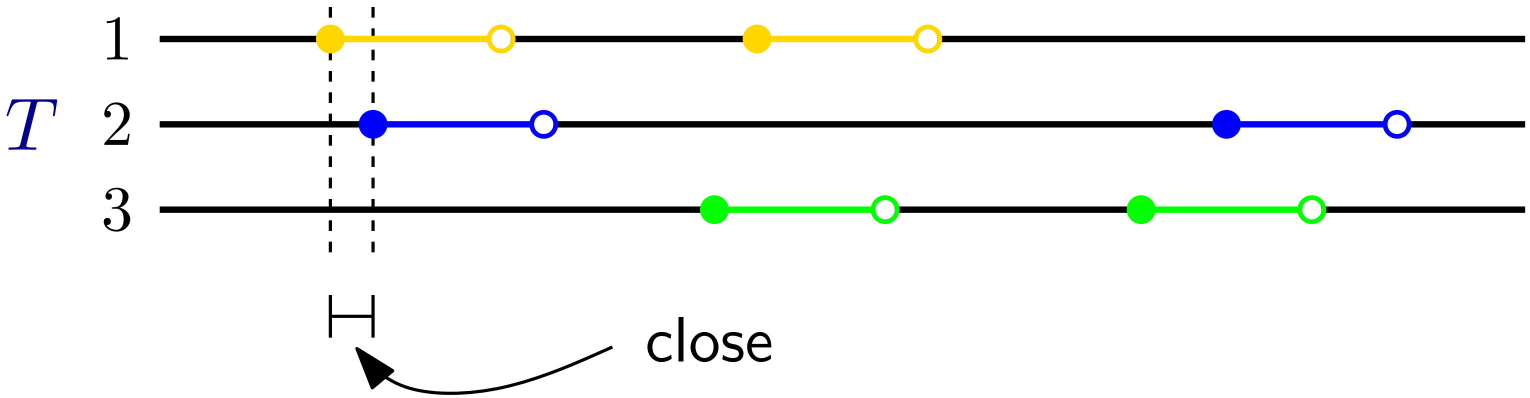

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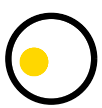

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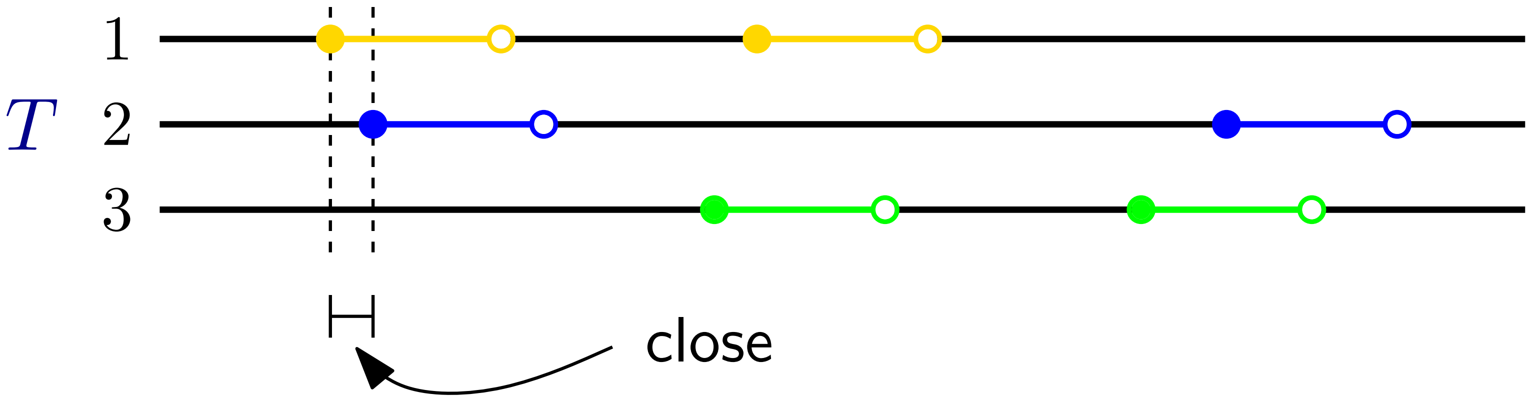

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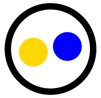
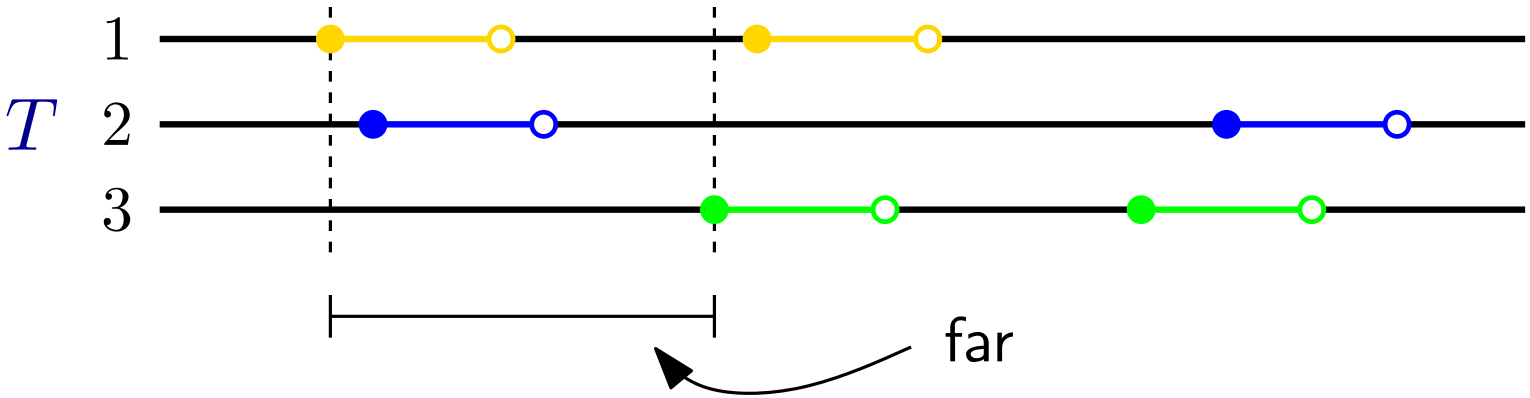
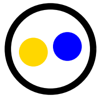


The overall idea



- We build a graph which encodes the structure of the queries



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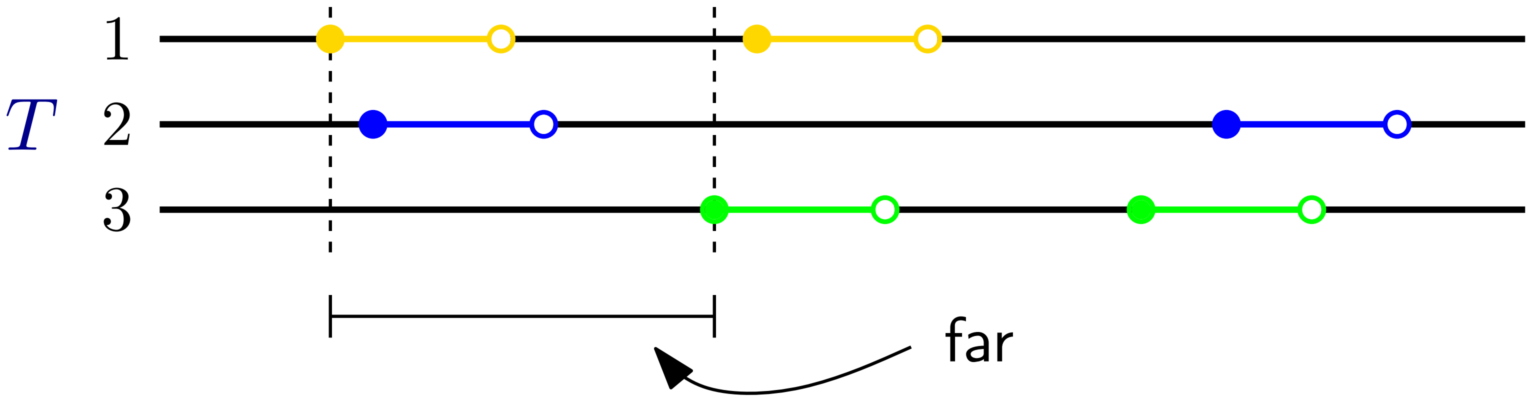

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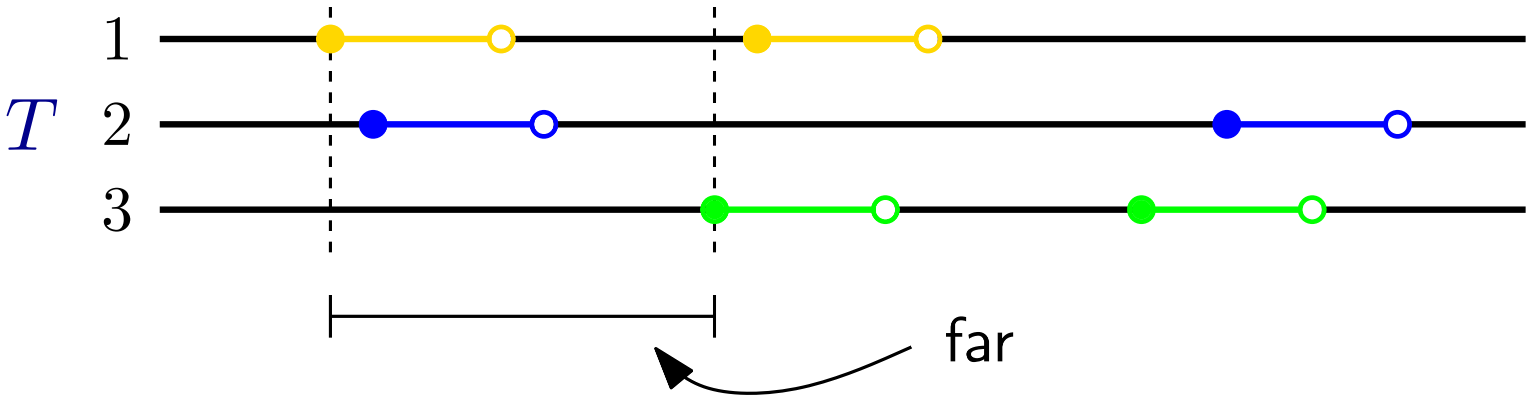

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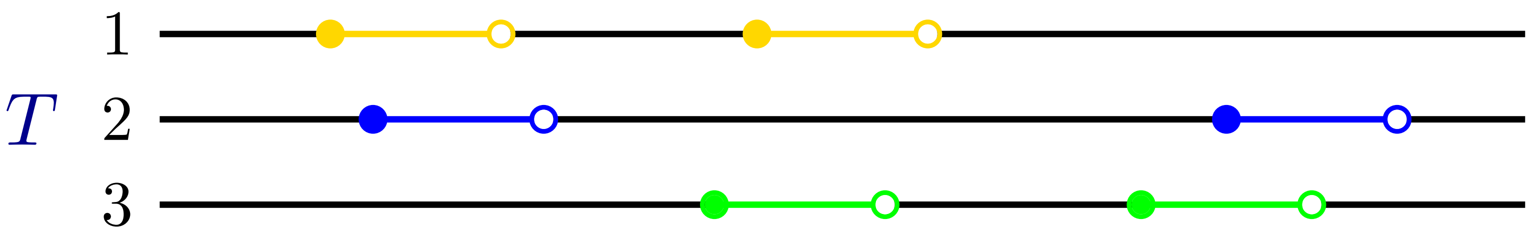

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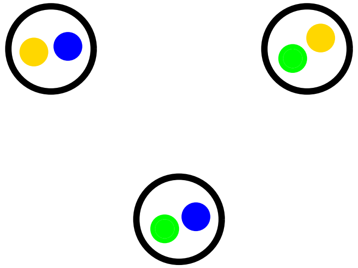

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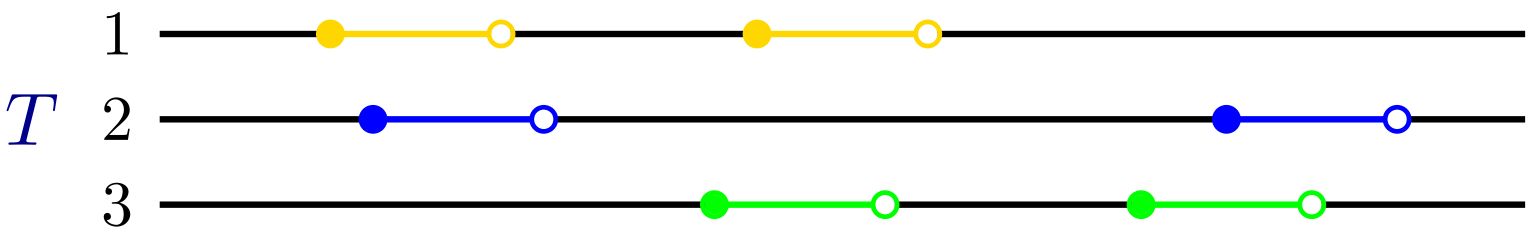

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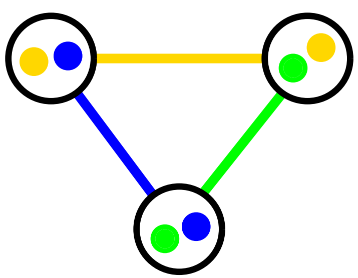

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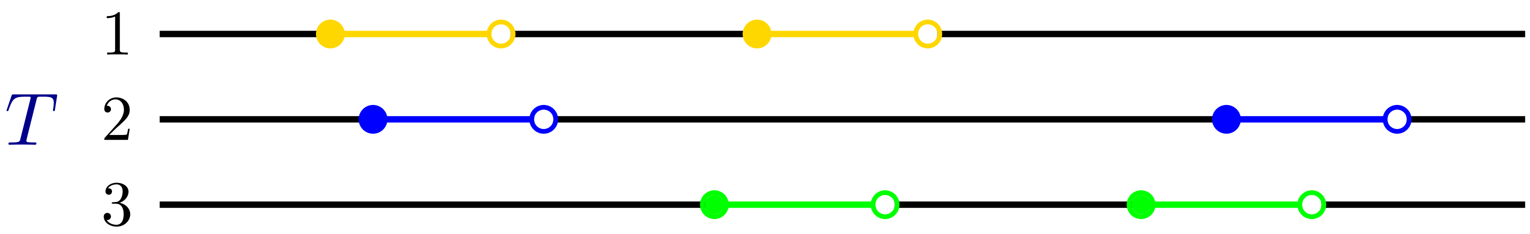

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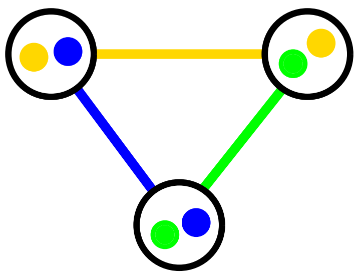
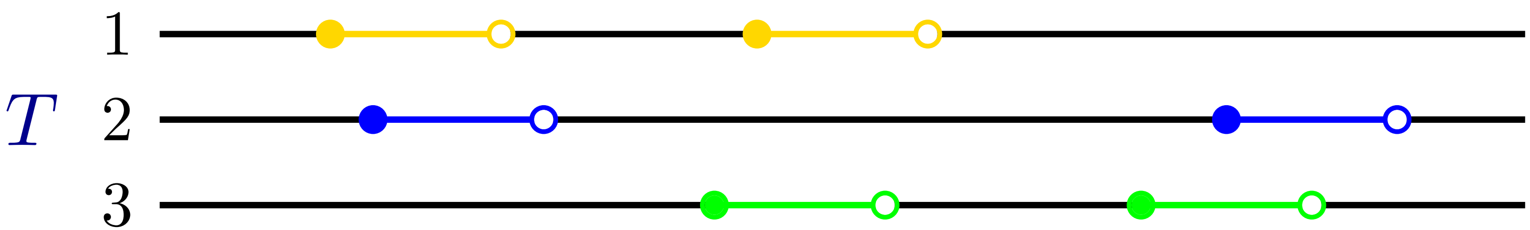


- We can apply one of the two tricks to any *short* cycle

The overall idea



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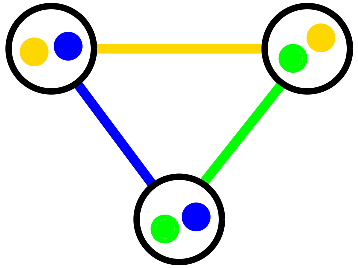
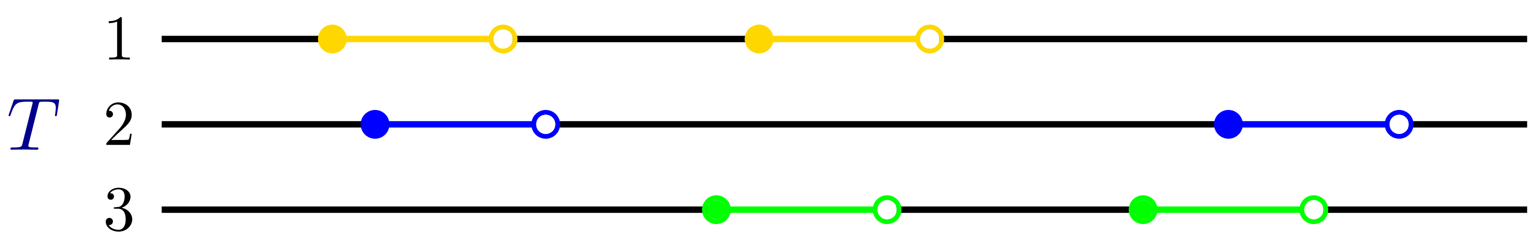


- We can apply one of the two tricks to any *short* cycle (length at most $2\log b + 1$)

The overall idea



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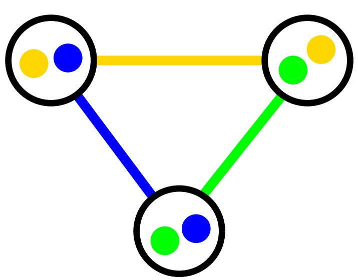
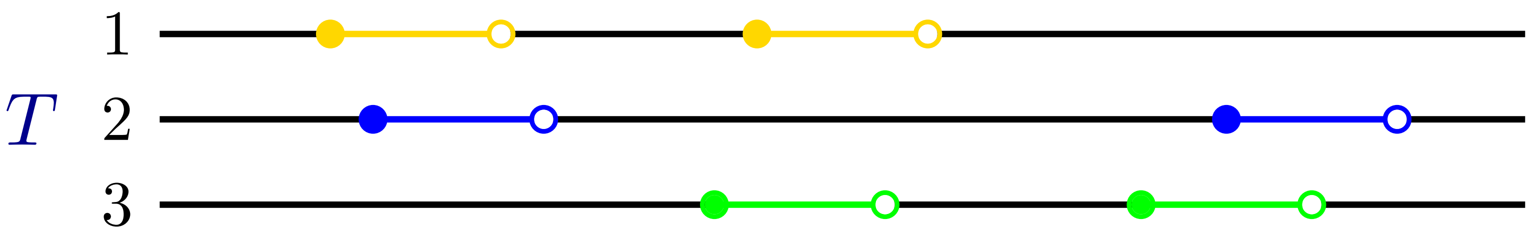


- We can apply one of the two tricks
  to any *short* cycle (length at most $2\log b + 1$)

- This breaks the cycle (because we delete an edge)

The overall idea



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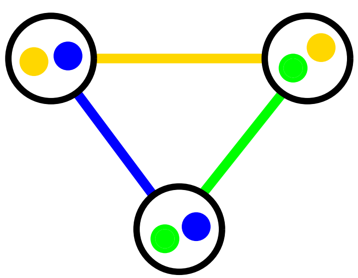

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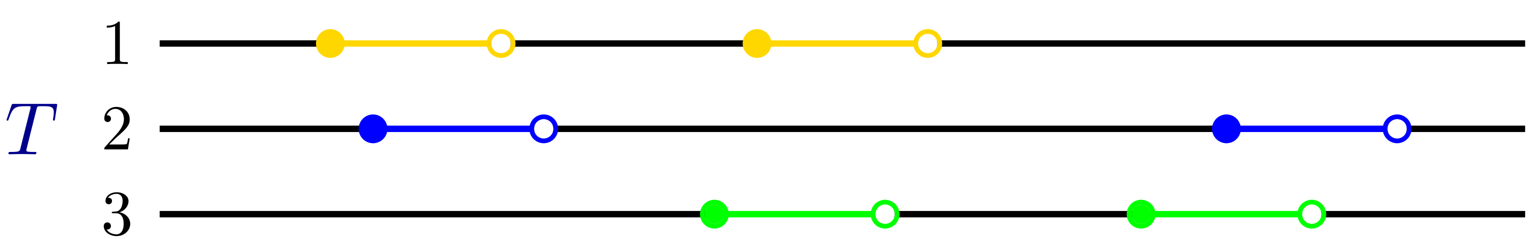
Fact If every node has degree at least three there is a short cycle

The overall idea

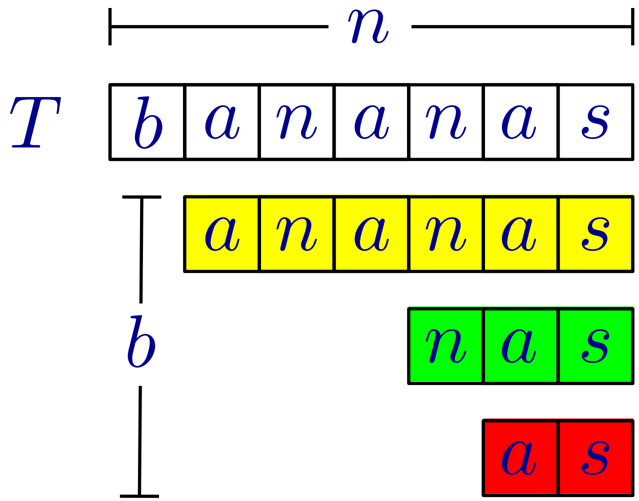


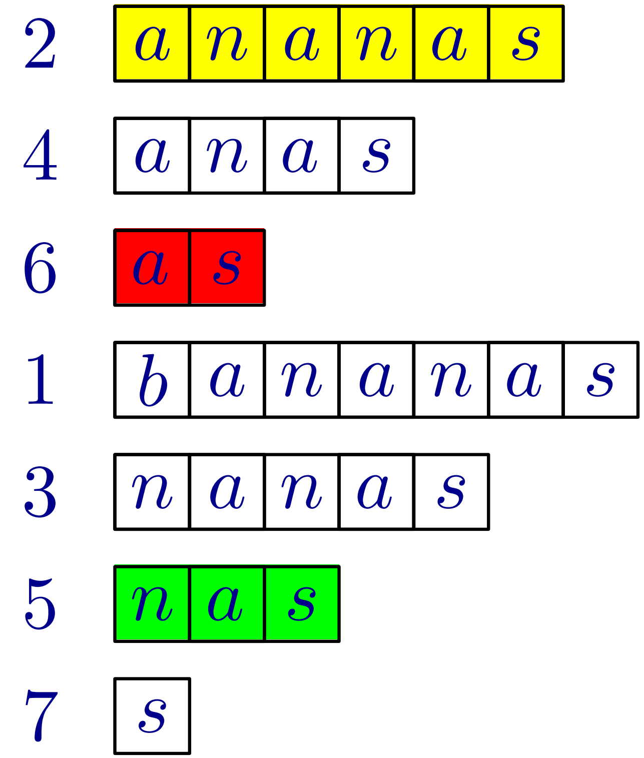
- We build a graph which encodes the structure of the queries

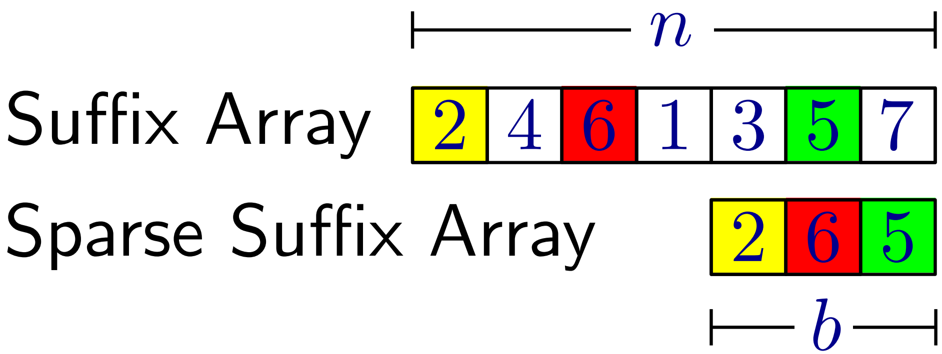
Fact If every node has degree at least three there is a short cycle

- Finding a short cycle in the graph takes $O(b)$ time
- This gives the additive $O(b^2 \log b)$ term
- All other steps take $O(n \log b)$ time over all rounds
  (and use $O(b)$ space)

Summary







- $O(n \log^2 b)$ time (Monte-Carlo)
- $O((n + b^2) \log^2 b)$ time with high probability (Las-Vegas)
- both in $O(b)$ space