String Indexing for Patterns with Wildcards

Philip Bille[1], Inge Li Gørtz[1], Hjalte Wedel Vildhøj[1], and Søren Vind[1]

[1]Technical University of Denmark, DTU Informatics

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DTU

String Indexing for Patterns with Wildcards

Problem Definition

Build an index for a string $t \in \Sigma^*$, that, given a query pattern p, quickly can report where p occurs in t.

$$p = p_0 \ * \ p_1 \ * \ \ldots \ * \ p_j$$

Example

$$t = \texttt{combinatorialpatternmatching}$$

$$p = * \texttt{ at } * * * \texttt{ n}$$

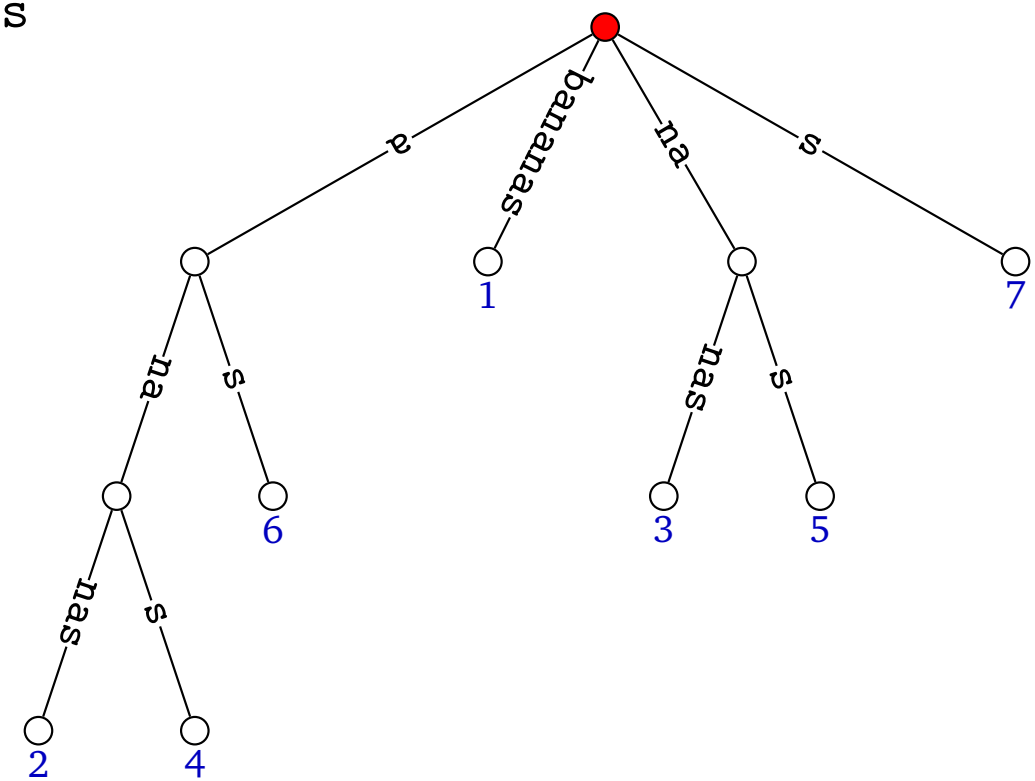
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
c	o	m	b	i	n	a	t	o	r	i	a	l	p	a	t	t	e	r	n	m	a	t	c	h	i	n	g
													*	a	t	*	*	*	n								
																				*	a	t	*	*	*	n	

Two Simple Solutions

Suffix Tree Search

$p = *\texttt{na}*$

$t = \overset{1\ 2\ 3\ 4\ 5\ 6\ 7}{\texttt{bananas}}$



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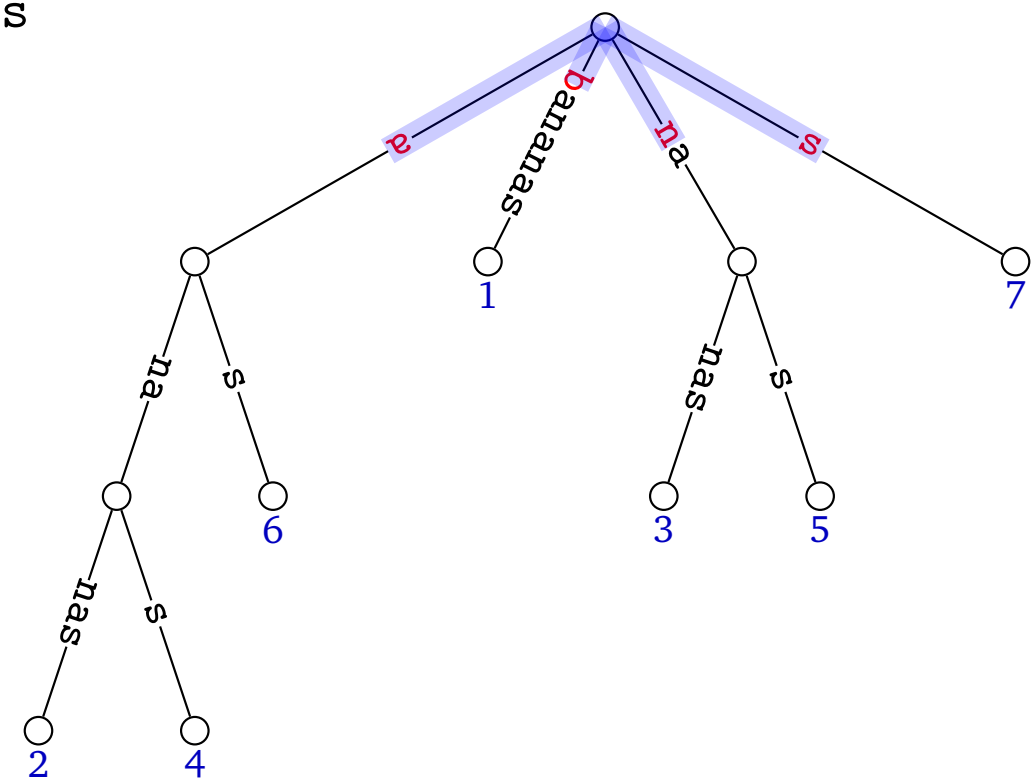

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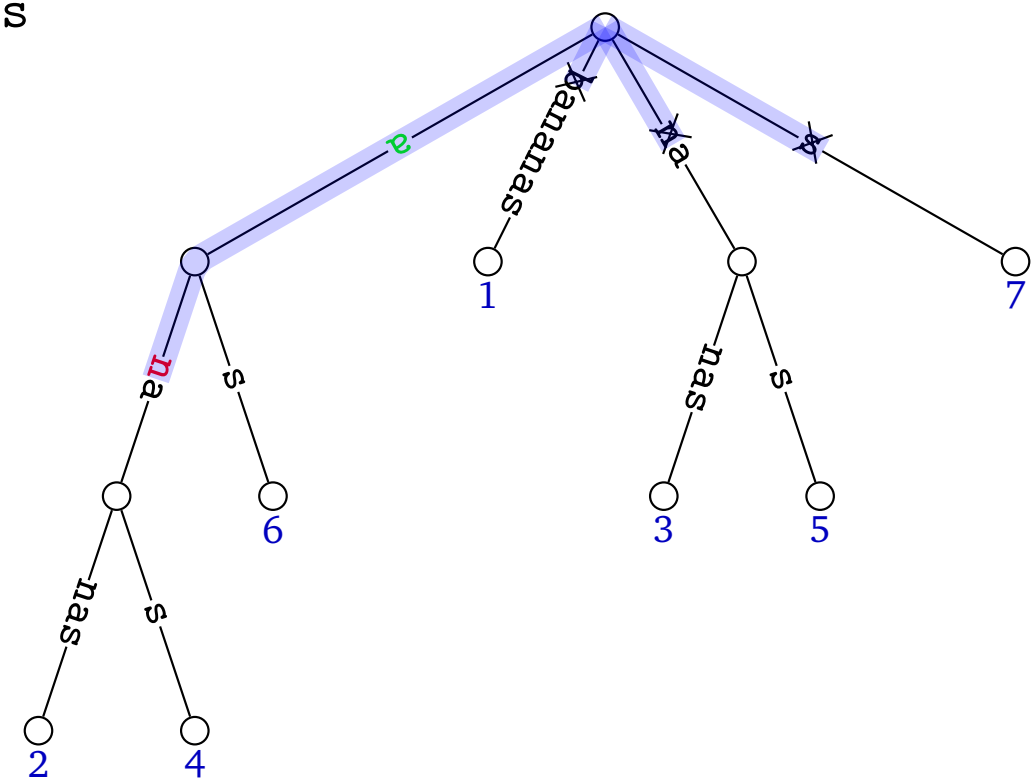

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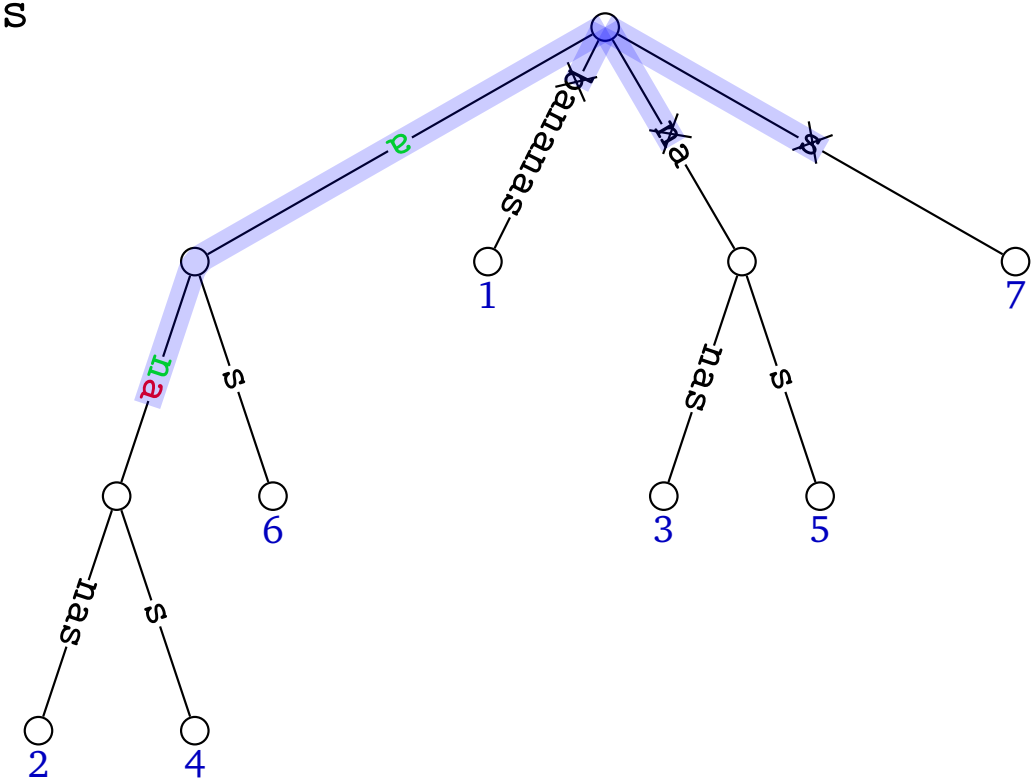

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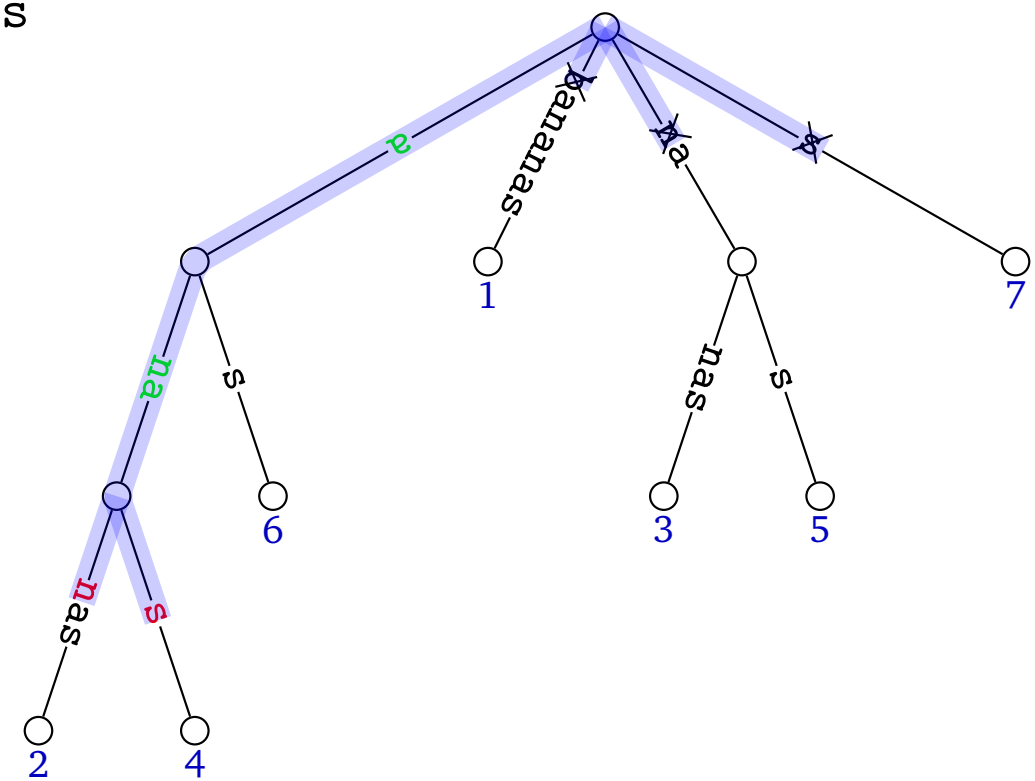

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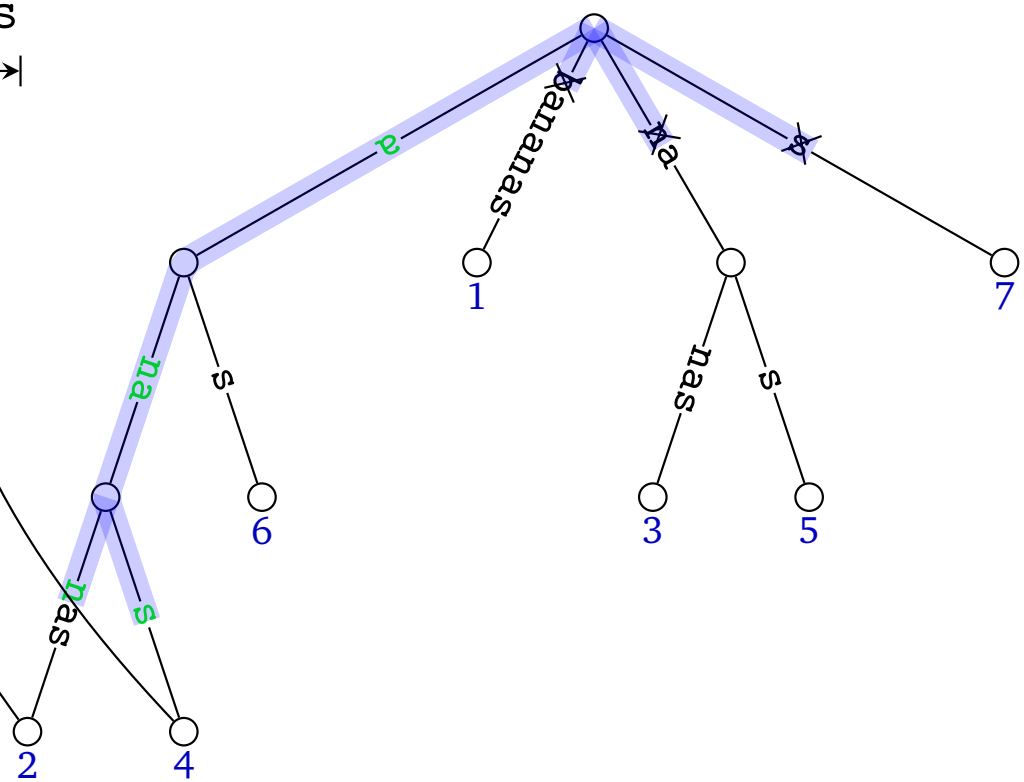

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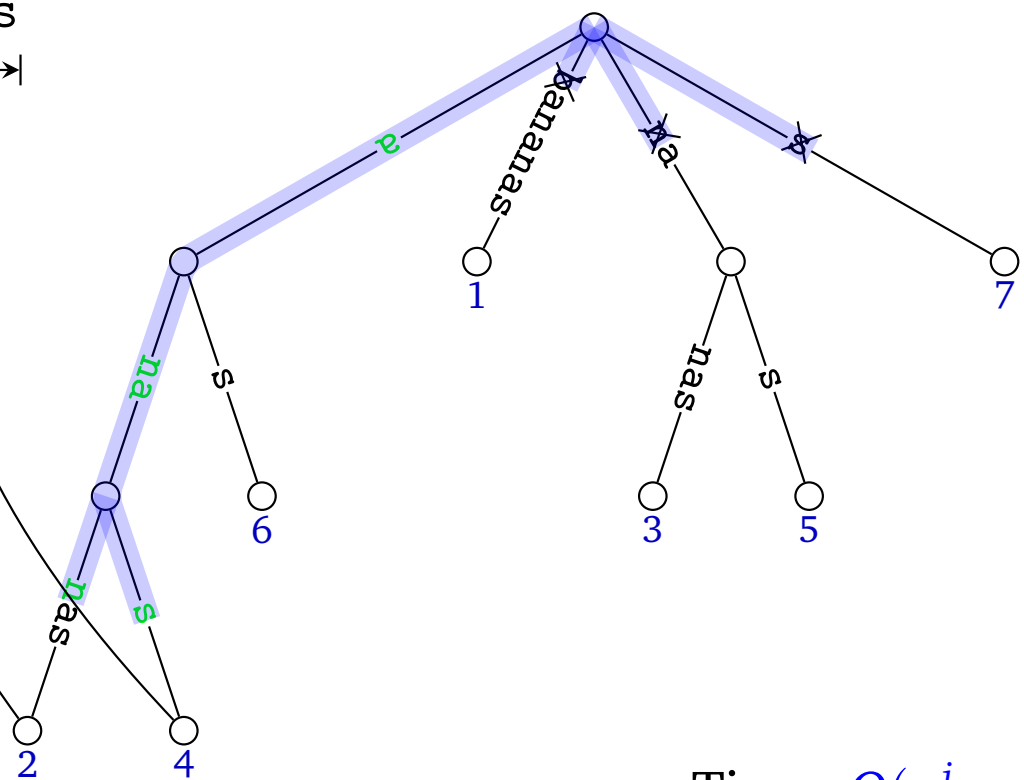
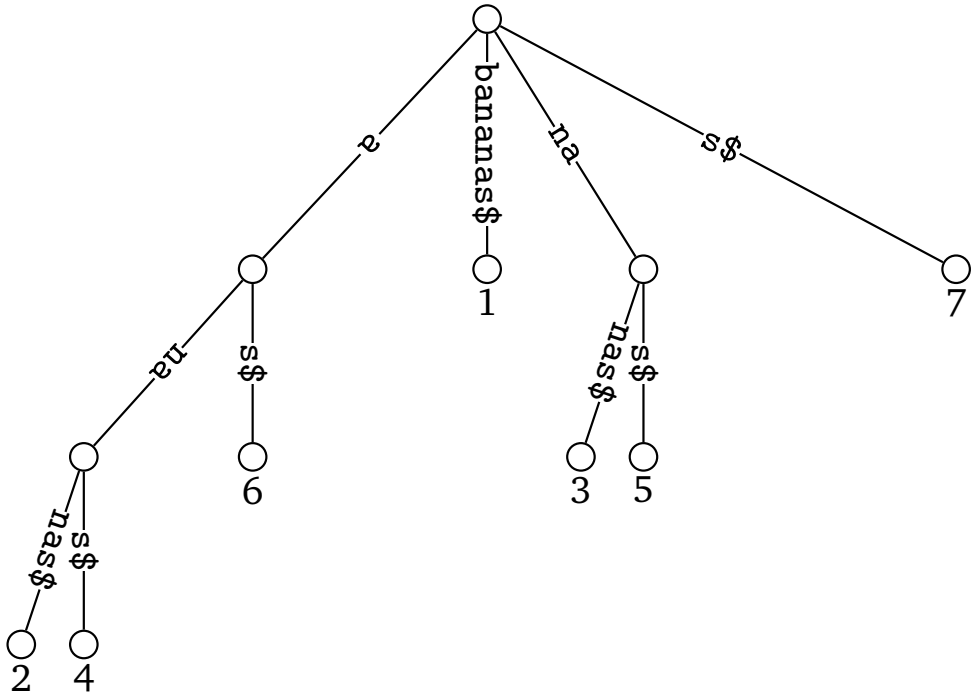


Time: $O(\sigma^j m + occ)$

Space: $O(n)$

Two Simple Solutions

Simple Linear Time Index



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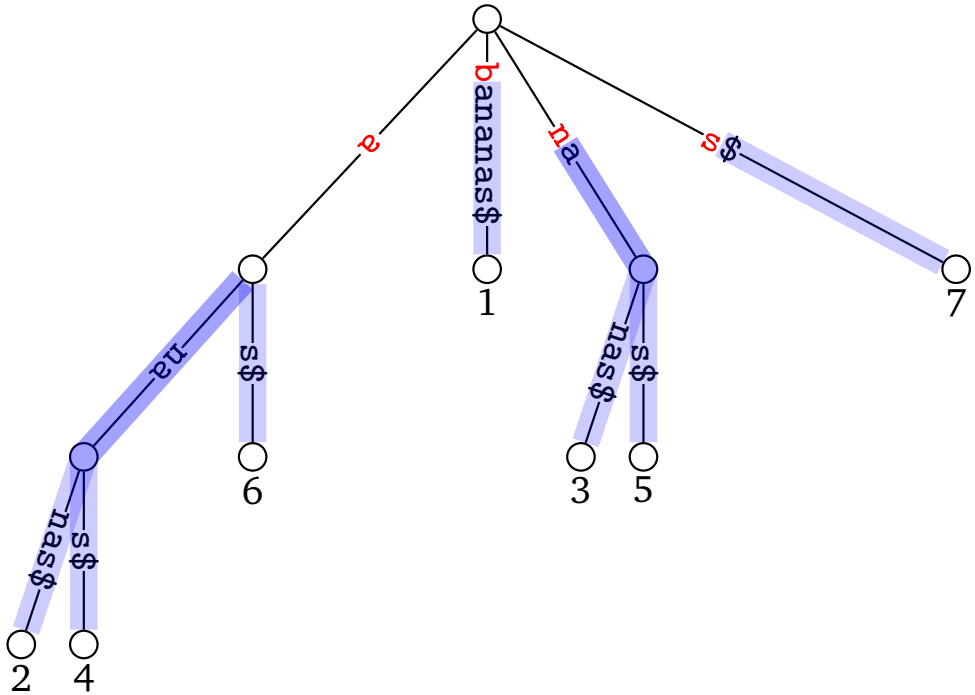

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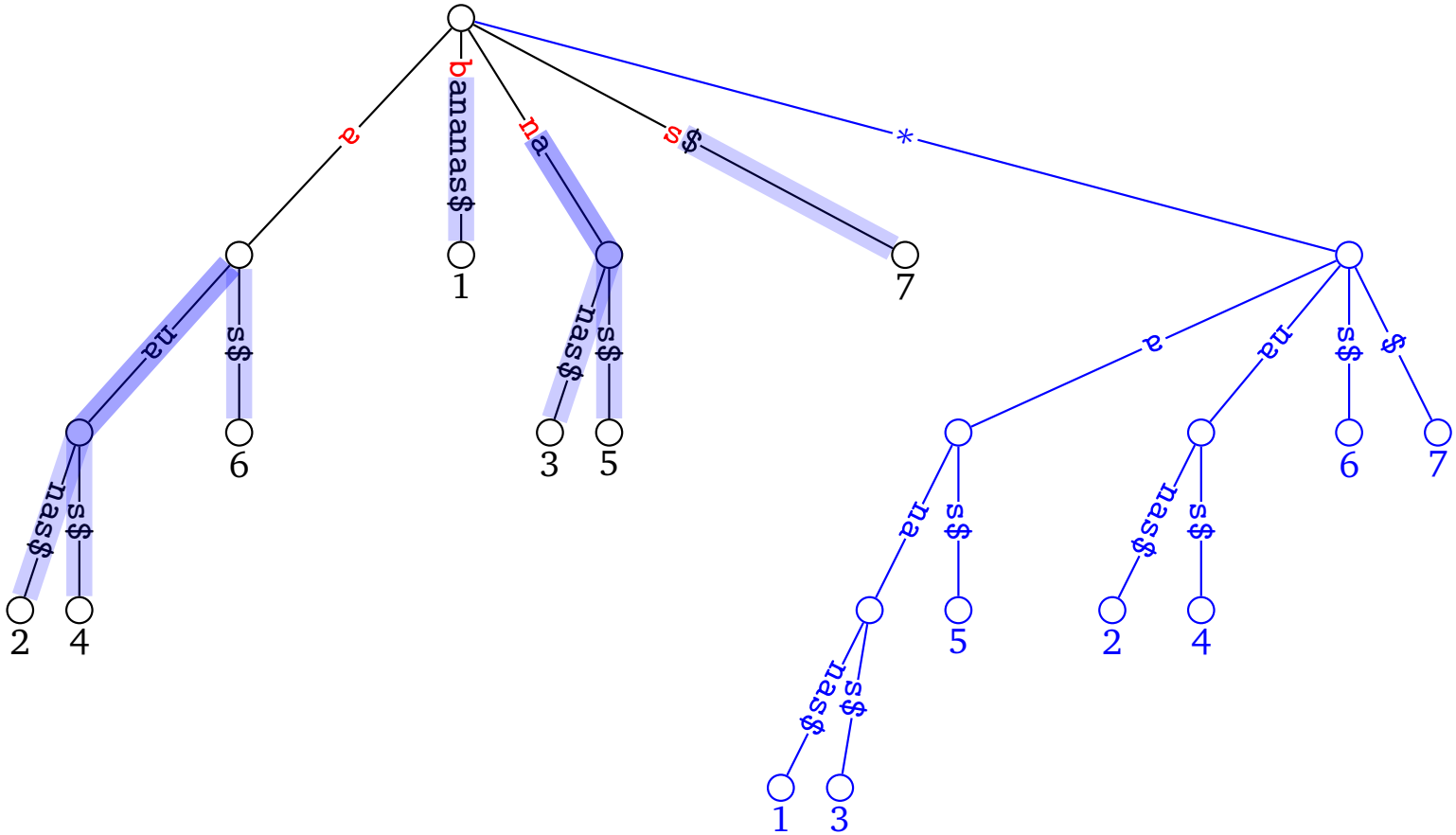

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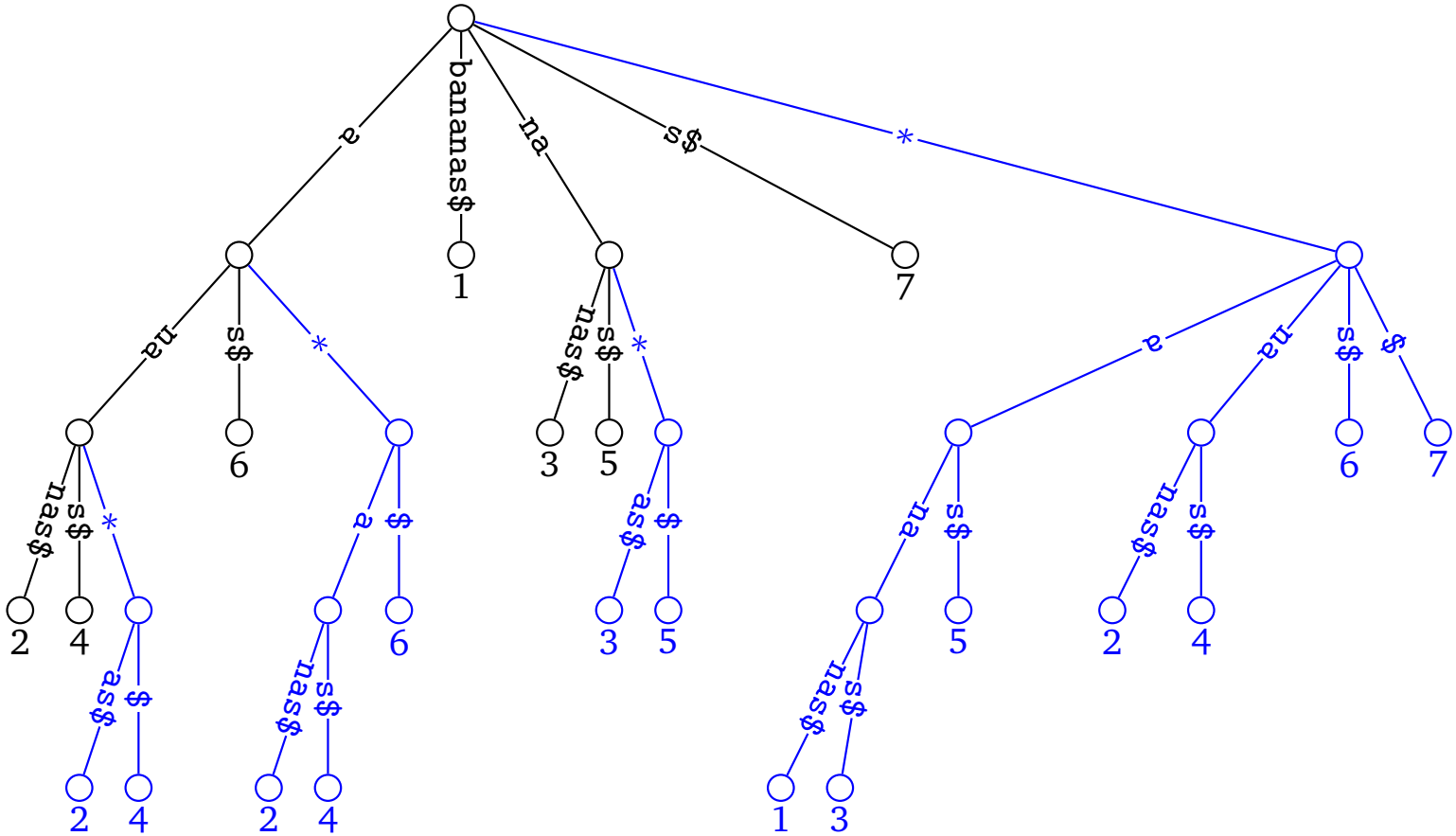

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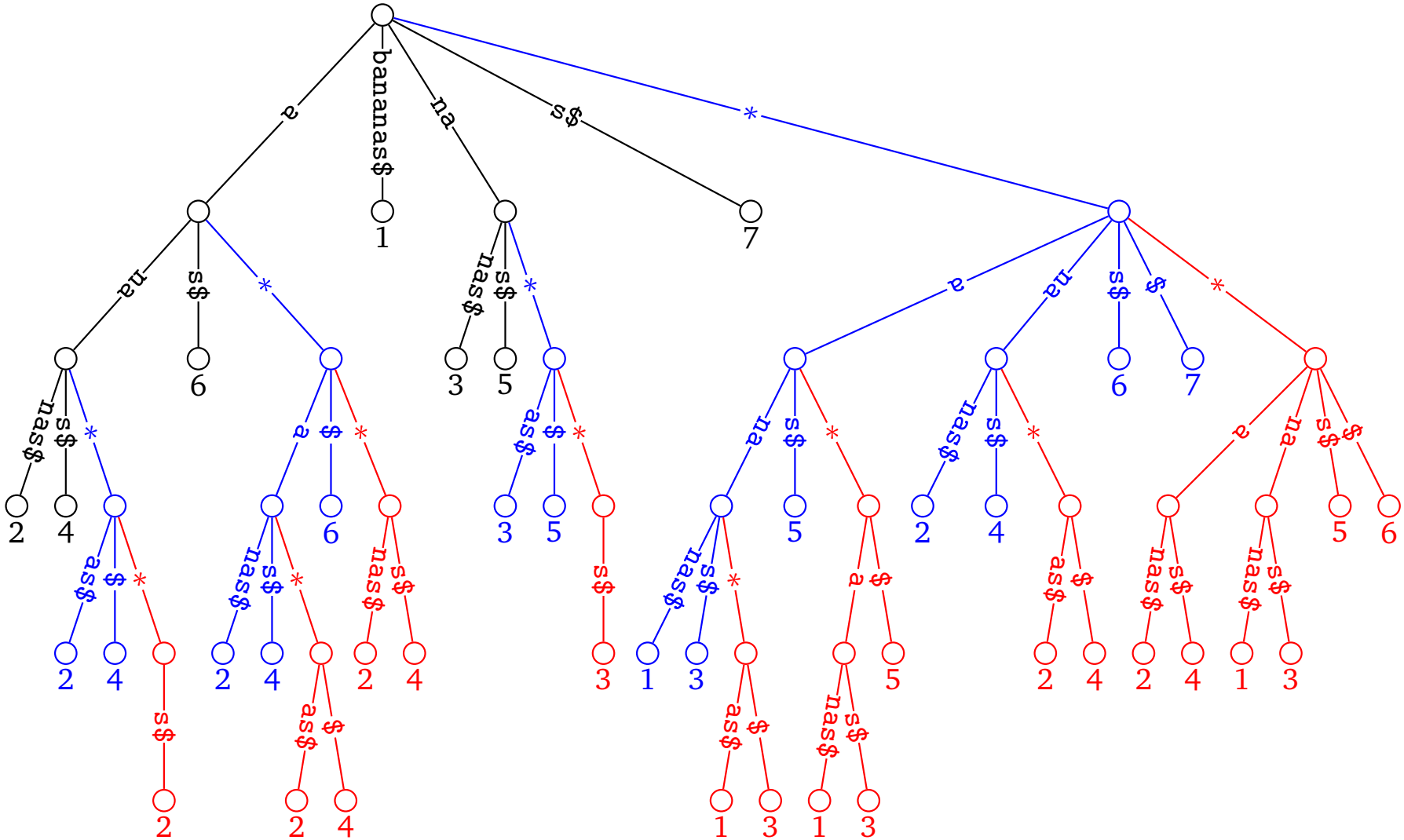

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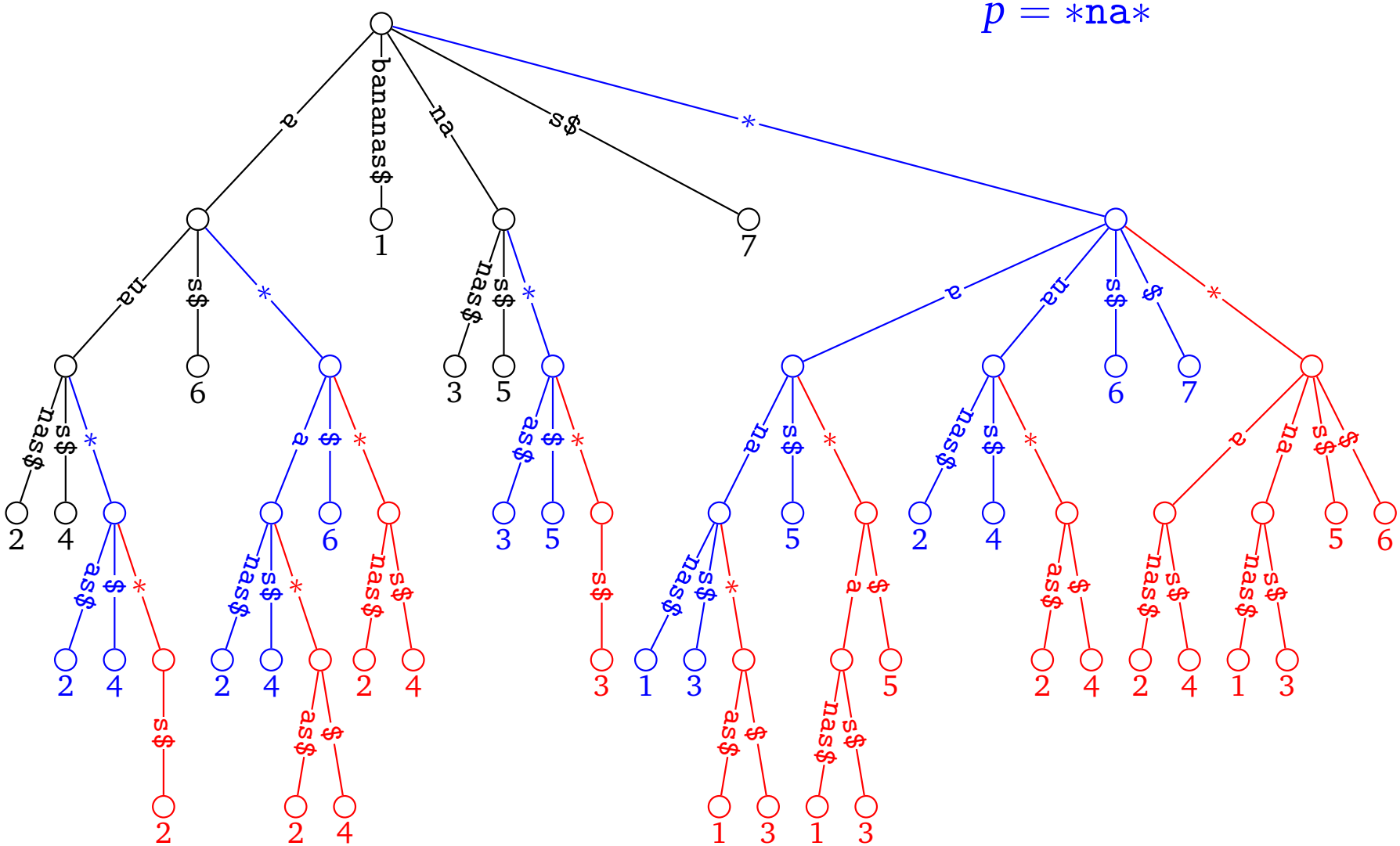

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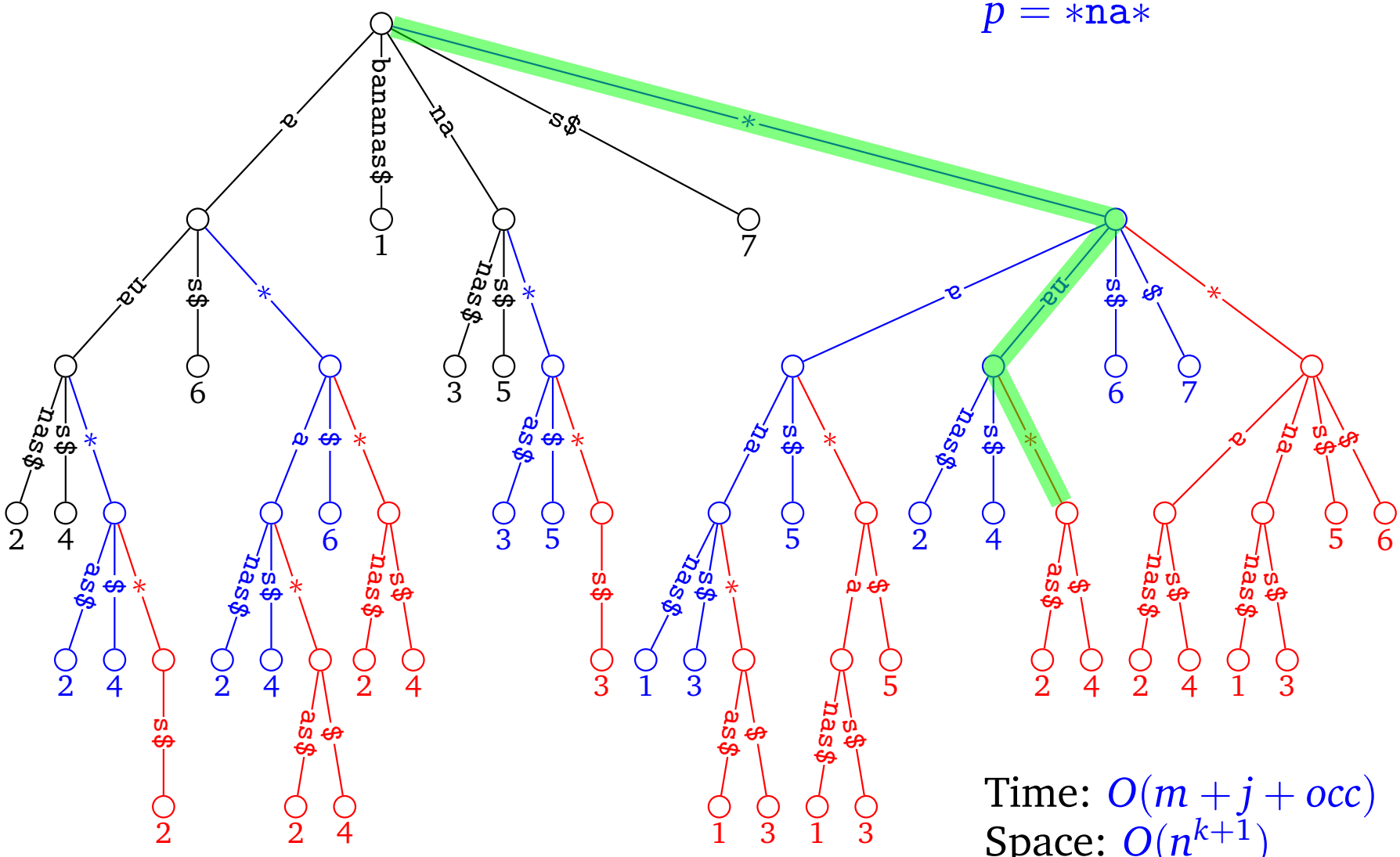


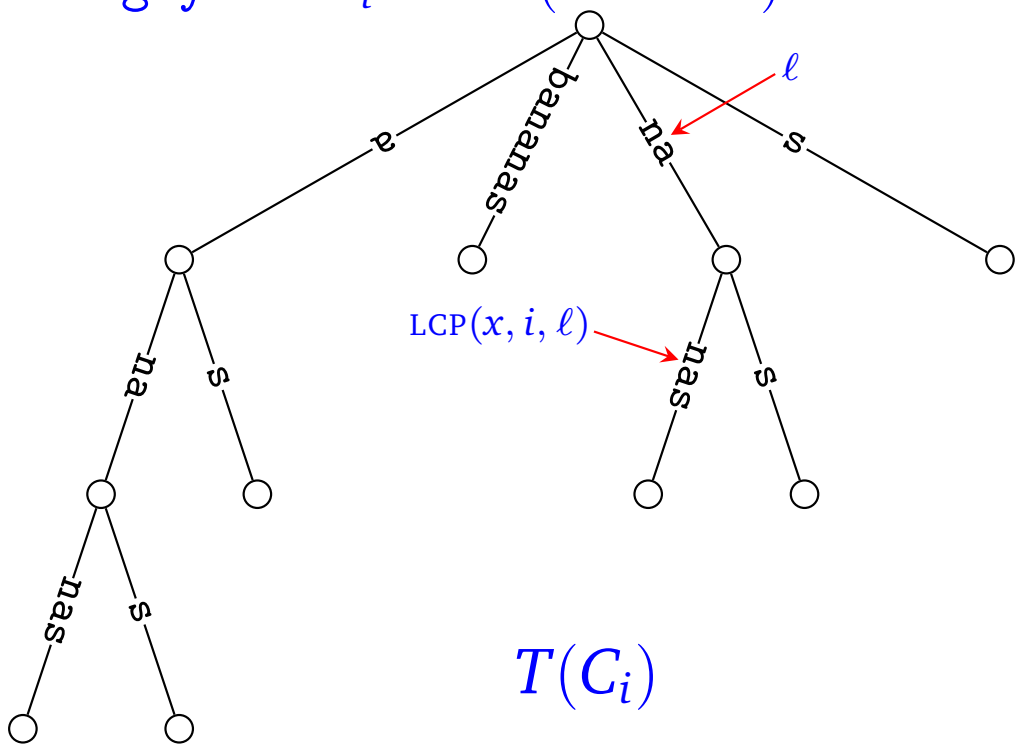
The Longest Common Prefix Data Structure [1]

LCP Queries

Let C_i be a set of substrings of the indexed string. Consider the following query on the compressed trie $T(C_i)$ storing the strings in C_i.

> $\text{LCP}(x, i, \ell)$: *The location where the search for $x \in \Sigma^*$ stops when starting in location $\ell \in T(C_i)$.*

Example: $x = \texttt{angry}$ and $C_i = \text{suff}(\texttt{bananas})$.



$T(C_i)$

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The Longest Common Prefix Data Structure [1]

An Application

Search for subpatterns in the suffix tree using the LCP data structure:

- Build the LCP data structure for the suffix tree.
- Search with a query pattern containing wildcards:
  - Search for complete subpatterns using LCP queries.
  - Branch on a wildcard as in the simple suffix tree solution.

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How fast can you answer an LCP query?

- $O(\log \log n)$ time and $O(n \log n)$ space.

  $\Rightarrow$ Index with query time $O(m + \sigma^j \log \log n + occ)$ and space $O(n \log n)$.

- We show that you can also do $O(\log n)$ time and $O(n)$ space.

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SOLUTION 1

An Unbounded Wildcard Index Using Linear Space

Query Time:	$O(m + \sigma^j \log\log n + occ)$
Space Usage:	$O(n)$

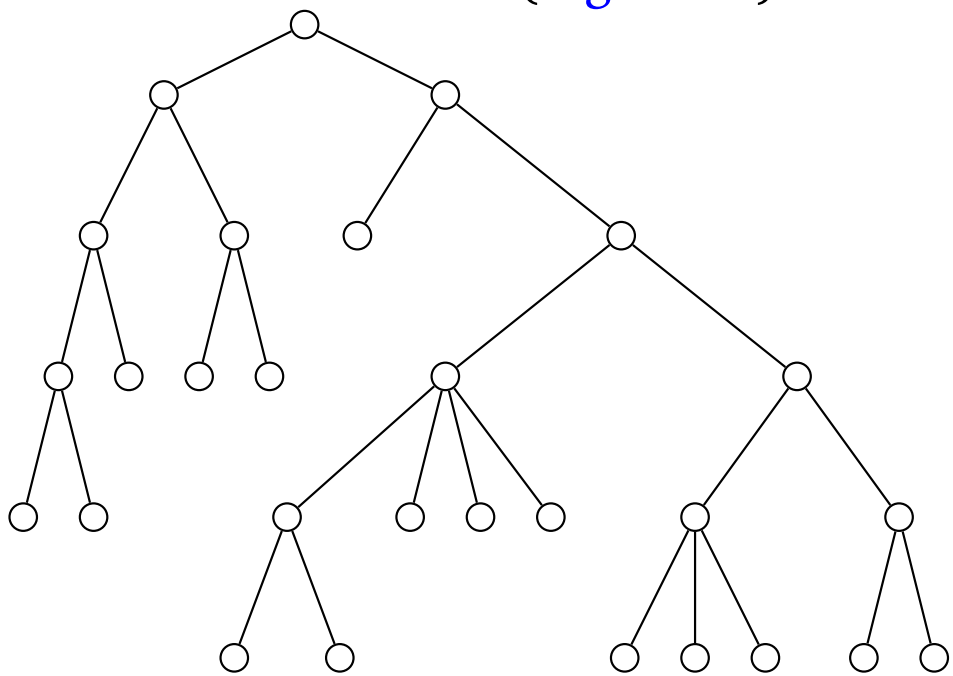
An Unbounded Wildcard Index Using Linear Space

ART Decomposition [2]

Definition:

- A *bottom tree* is a maximal subtree with at most $\log n$ leaves.
- Vertices not in a bottom tree constitute the *top tree*.

Example: A tree with $n = 16$ leaves ($\log n = 4$).



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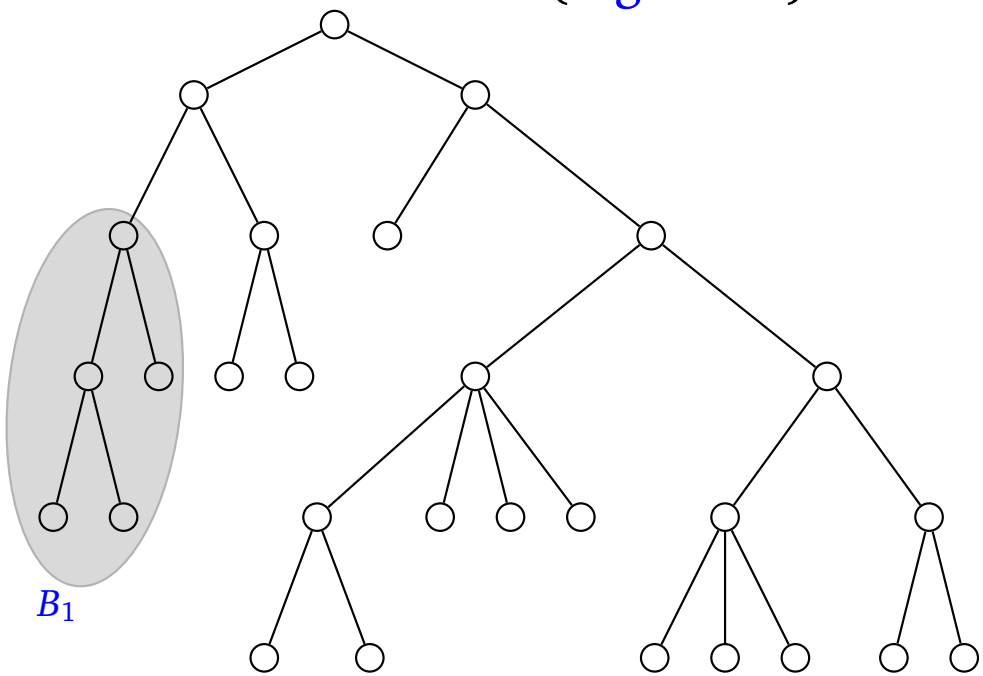

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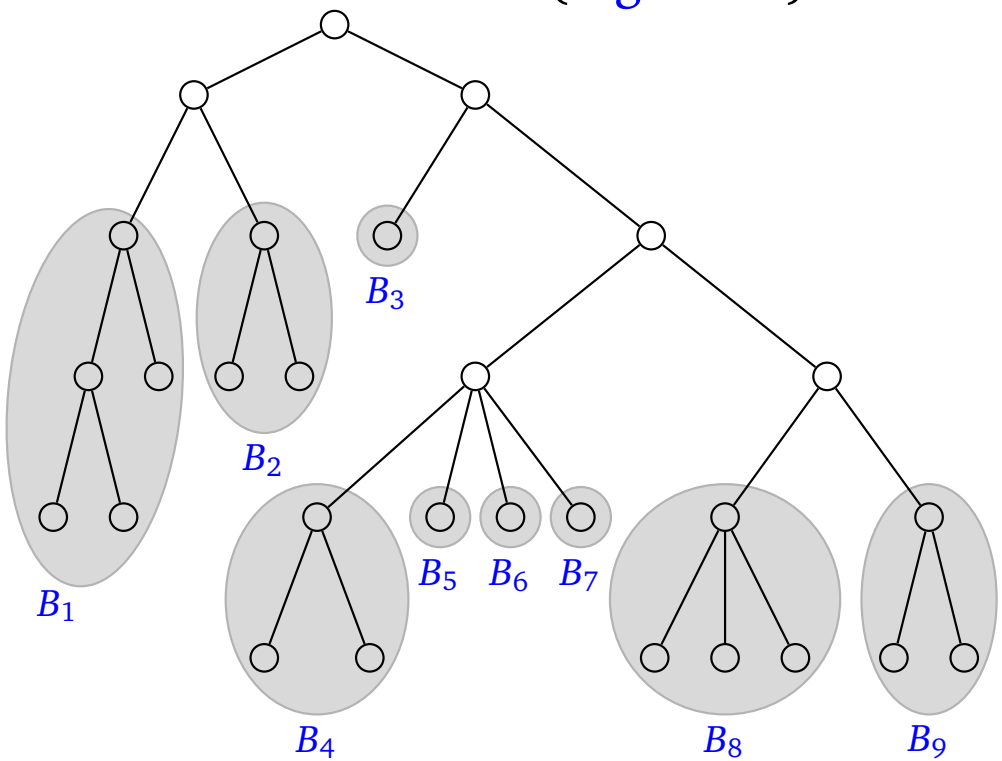

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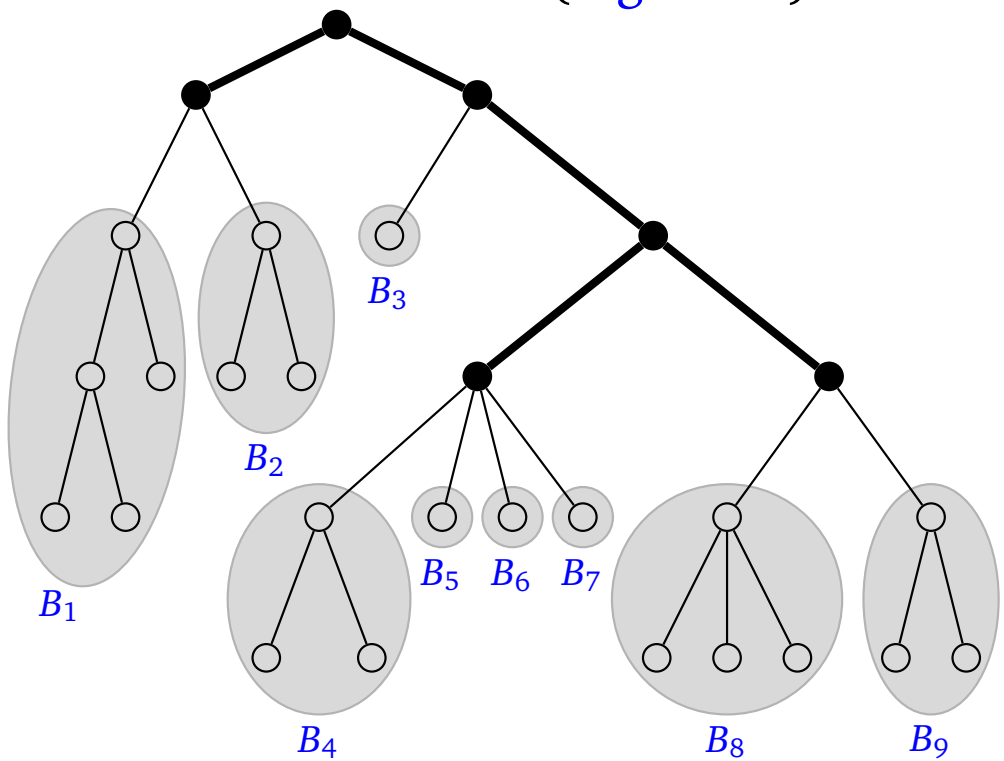

Property: The top tree has $O(\frac{n}{\log n})$ leaves.

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An Unbounded Wildcard Index Using Linear Space

Obtaining the Index

- Use the ART decomposition to decompose the suffix tree into a number of logarithmic sized bottom trees and a single top tree containing $O(\frac{n}{\log n})$ leaves.
- Store the top and bottom trees in LCP data structure.
- On the top tree T': Add support for $O(\log \log n)$ time LCP queries using the method by Cole et al. [3]
  - This requires space $O(|T'| \log |T'|) = O(\frac{n}{\log n} \log(\frac{n}{\log n})) = O(n)$.
- On the bottom trees $T(C_1), \ldots, T(C_q)$: Add support for $O(\log n)$ time LCP queries using our new method.
  - This requires $O(\sum_{i=1}^{q} |C_i|) = O(n)$ space.
  - The query time becomes $O(\log |C_i|) = O(\log \log n)$.

This gives an unbounded wildcard index using $O(n)$ space with query time $O(m + \sigma^j \log \log n + occ)$.

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SOLUTION 2

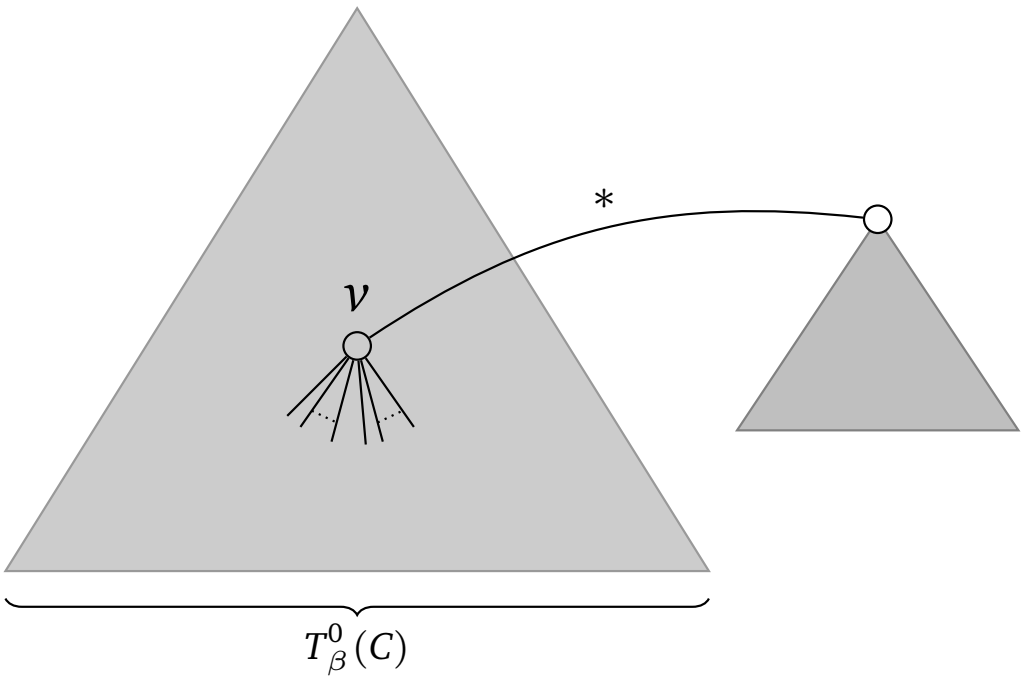
A Time-Space Trade-Off for k-Bounded Wildcard Indexes

Query Time:	$O(m + \beta^j \log\log n + occ)$
Space Usage:	$O(n \log_\beta^{k-1}(n) \log n)$

A Time-Space Trade-Off for Bounded Wildcard Indexes

General Idea

Reduce the branching factor of the suffix tree search from σ to β by creating wildcard trees. Query time: $O(m + \beta^j \log\log n + occ)$ when using the LCP data structure.



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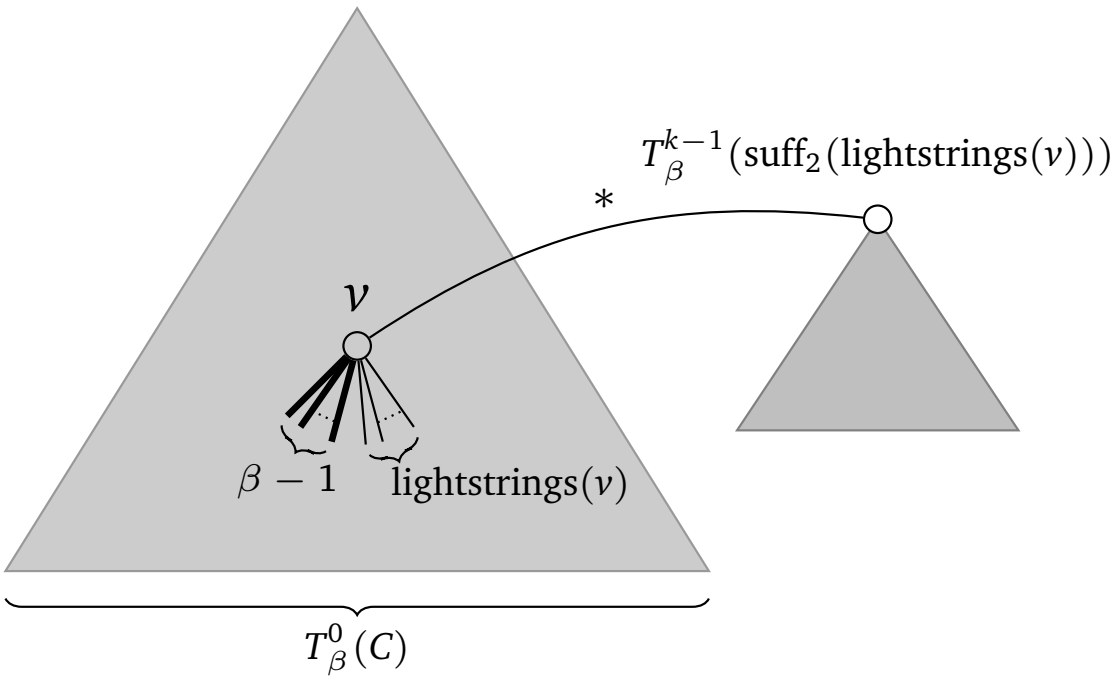

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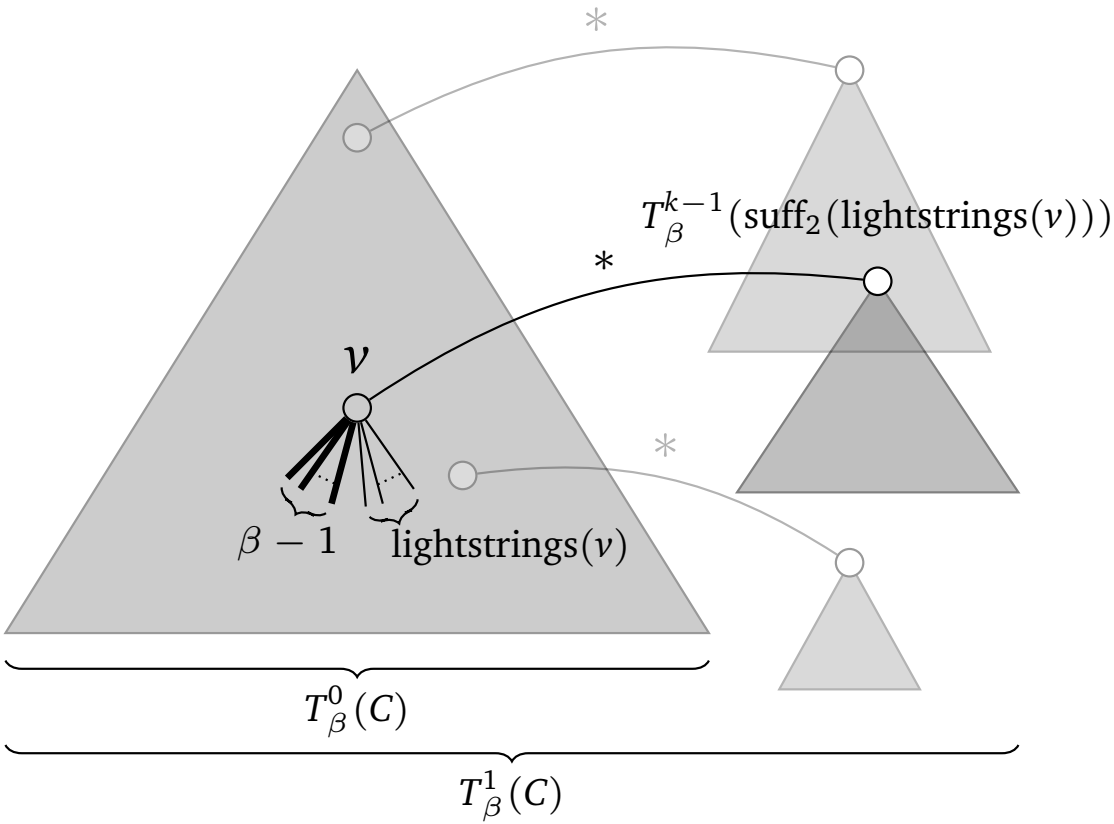

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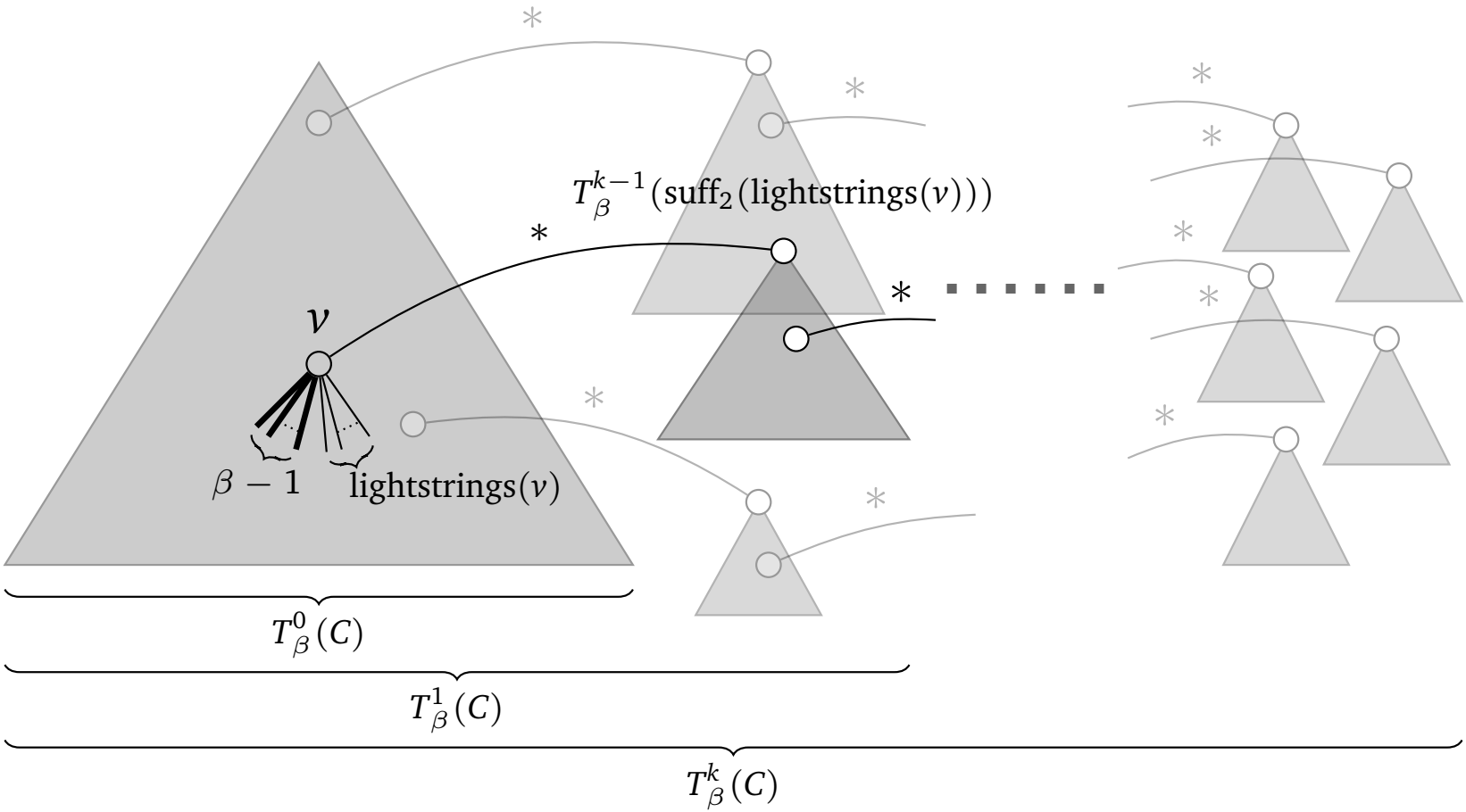
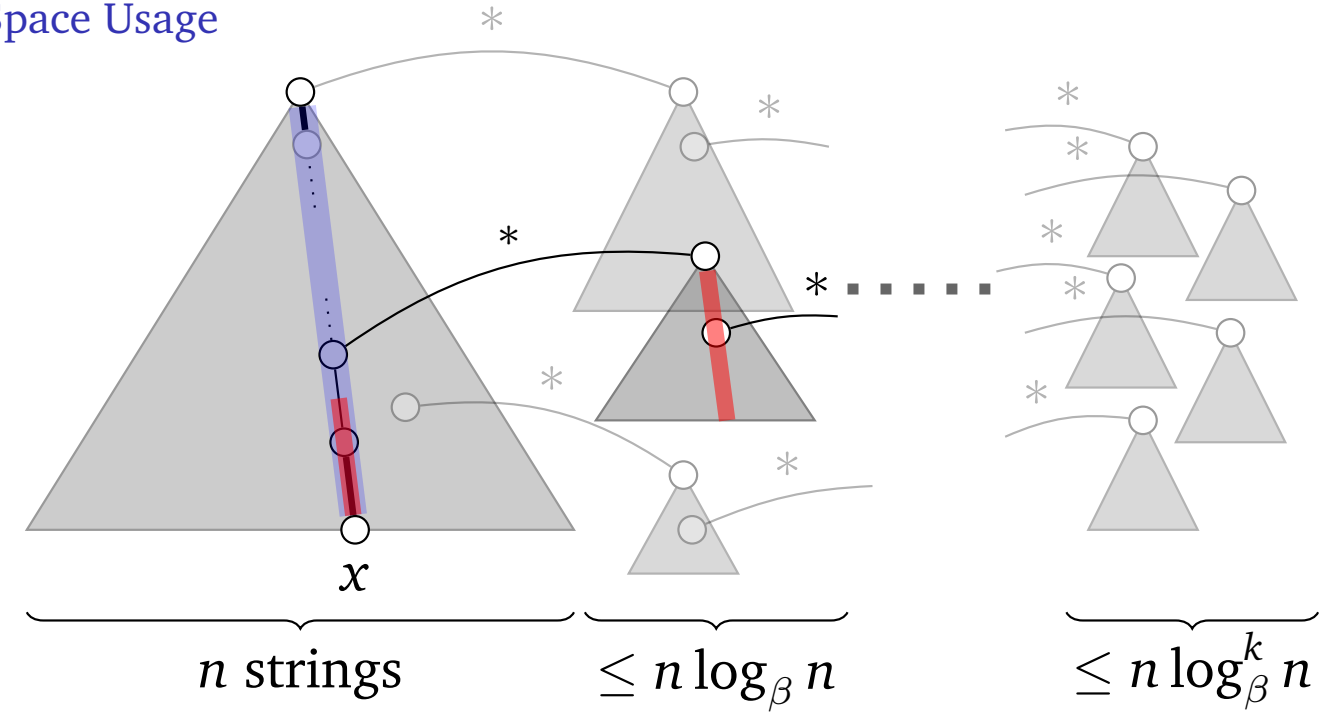


A Time-Space Trade-Off for Bounded Wildcard Indexes

Analysing the Space Usage



Each string in $T(C)$ gives rise to at most $\text{lightdepth}(x) \leq \log_\beta n$ strings on the next level. So the number of strings in a k-level index is at most

$$\sum_{i=0}^{k} n \log_\beta^i n = O(n \log_\beta^k n) \ .$$

By using the LCP data structure to support LCP queries on every subtrie, we obtain a k-bounded wildcard index with query time $O(m + \beta^j \log\log n + occ)$ using space $O(n \log_\beta^{k-1}(n) \log n)$.

SOLUTION 3

A k-Bounded Wildcard Index with Linear Query Time

Query Time:	$O(m + j + occ)$
Space Usage:	$O(n\sigma^{k^2} \log^k \log n)$

A k-Bounded Wildcard Index with Linear Query Time

General Idea

Consider the previously described unbounded wildcard index $\mathcal{A}$ with

- linear space usage, and
- query time $O(m + \sigma^j \log\log n + occ)$.

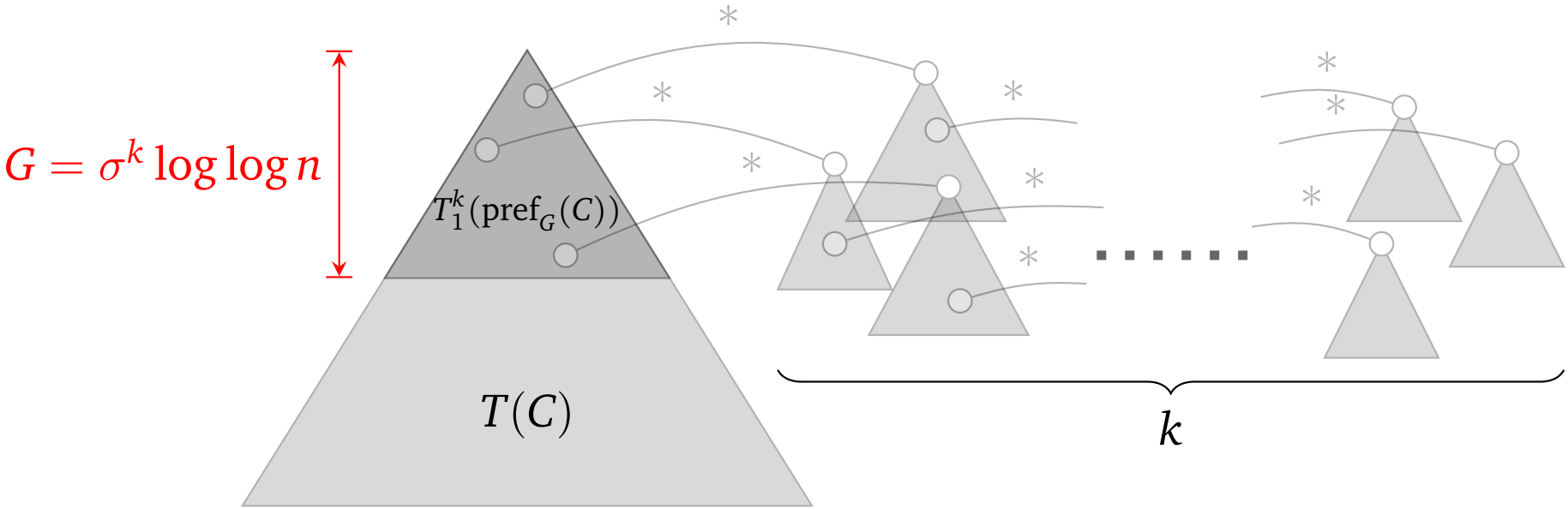
Suppose the pattern is restricted to contain a maximum of k wildcards.

- If $m + j > \sigma^k \log\log n > \sigma^j \log\log n$, (i.e., the query pattern is long) the query time becomes linear: $O(m + j + occ)$.
- If $m + j \leq \sigma^k \log\log n$, we query a special wildcard index $\mathcal{B}$ for short patterns with query time $O(m + j + occ)$.

In any case the query time is $O(m + j + occ)$. The space used by the index is $O(|\mathcal{A}| + |\mathcal{B}|)$.

A k-Bounded Wildcard Index with Linear Query Time

A Special Index for Patterns Shorter than $\sigma^k \log\log n$



$T_1^0(\text{pref}_G(C))$ contains at most n strings. Consider a string x in one of the subtries. At most $|x| \leq G$ suffixes of x appear in tries on the next level. Consequently, the number of strings in $T_1^k(\text{pref}_G(C))$ is bounded by

$$\sum_{i=0}^{k} nG^i = O(n(\sigma^k \log\log n)^k) = O(n\sigma^{k^2} \log^k \log n) \ .$$

Result: A k-bounded wildcard index with linear query time $O(m + j + occ)$ using space $O(n\sigma^{k^2} \log^k \log n)$.

Conclusions

- Three new solutions for string indexing for patterns with wildcards:
  - The fastest linear space index.
  - A trade-off for k-bounded wildcard indexes.
  - The first non-trivial linear time index.
- All solutions generalize to string indexing for patterns with variable length gaps.

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Thank you!