String Matching with Variable Length Gaps

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Presented by Hjalte Wedel Vildhøj

October 13, 2010

SPIRE 2010, Los Cabos, Mexico

DTU Informatics

Department of Informatics and Mathematical Modelling

The Variable Length Gap Problem

Given some string $T \in \Sigma^+$ and a *variable length gap pattern*

$$P = P_1 \cdot \ g\{a_1, b_1\} \cdot P_2 \cdot g\{a_2, b_2\} \cdots g\{a_{k-1}, b_{k-1}\} \cdot P_k \ .$$

Find the *end positions* for all occurrences of P in T.

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Example: $P = \mathrm{A} \cdot g\{6, 7\} \cdot \mathrm{CC} \cdot g\{2, 6\} \cdot \mathrm{GT}$

$T =$ ATCGGCTCCAGACCAGTACCCGTTCCGTGGT

Solution: $\{\}$

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$$T = \mathrm{A}\overbrace{\mathrm{TCGGCT}}^{6}\mathrm{CC}\overbrace{\mathrm{AGACCA}}^{6}\mathrm{GT}\mathrm{ACCCGTTCCGTGGT}$$

end pos in T

Solution: $\{17\}$

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$$T = \mathrm{ATCGGCTCCA}\overbrace{\mathrm{GACCAGTA}}^{8}\mathrm{CC}\overbrace{\mathrm{CGTTCC}}^{6}\mathrm{GTGGT}$$

Solution: $\{17\}$

Not a valid match!

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$$T = \ \mathrm{ATCGGCTCCAGA}\,\mathrm{A}\overbrace{\mathrm{CCAGTA}}^{6}\mathrm{CC}\overbrace{\mathrm{CGTTCC}}^{6}\mathrm{GT}\,\mathrm{GGT}$$

end pos in T

Solution: $\{17, 28\}$

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$$T = \mathrm{ATCGGCTCCAGA}\,\mathrm{A}\overbrace{\mathrm{CCAGTAC}}^{7}\mathrm{CC}\overbrace{\mathrm{GTTCC}}^{5}\mathrm{GT}\,\mathrm{GGT}$$

end pos in T

Solution: $\{17, 28\}$

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$$T = \text{ATCGGCTCCAGACCAGTA}\overbrace{\text{CCCGTT}}^{7}\text{CC}\overbrace{\text{GTG}}^{3}\text{GT}$$

end pos in T

Solution: $\left\{17, 28, 31\right\}$

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A Closer Look At The Problem

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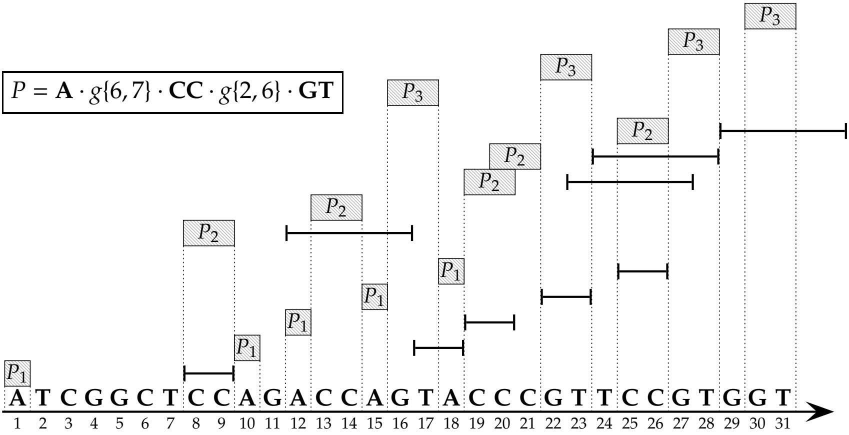

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Parameters

$n = |T|$

$\alpha = \#\text{ occ. of } P_1, P_2, \ldots, P_k \text{ in } T$

$$m = \sum_{i=1}^{k} |P_i|$$

$$A = \sum_{i=1}^{k} a_i$$

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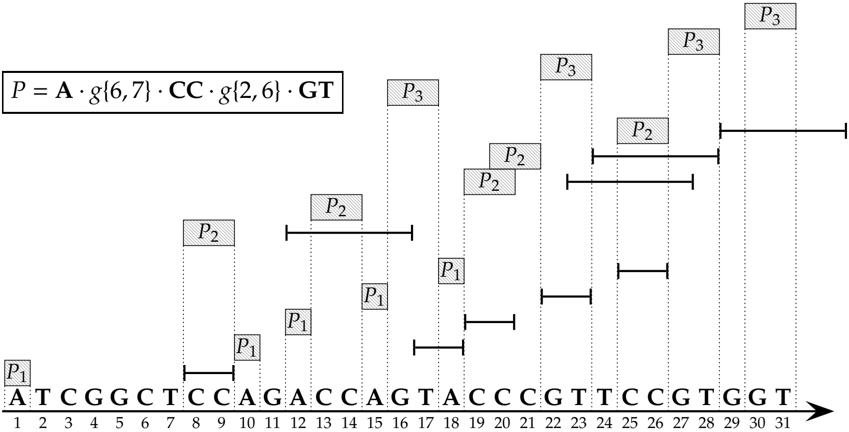

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Known Upper Bounds

By	*Time*	*Space*
Bille & Thorup[1]	$O\left(n(k\frac{\log w}{w} + \log k) + m \log m + A\right)$	$O(m + A)$
Morgante et al.[2]	$O((n + m) \log k + \alpha)$	$O(m + \alpha)$

[1] P. Bille and M. Thorup. Regular expression matching with multi-strings and intervals. In *Proc. 21st SODA, 2010*

[2] M. Morgante, A. Policriti, N. Vitacolonna, and A. Zuccolo. Structured motifs search. *J. Comput. Bio.*, 12(8):1065-1082, 2005

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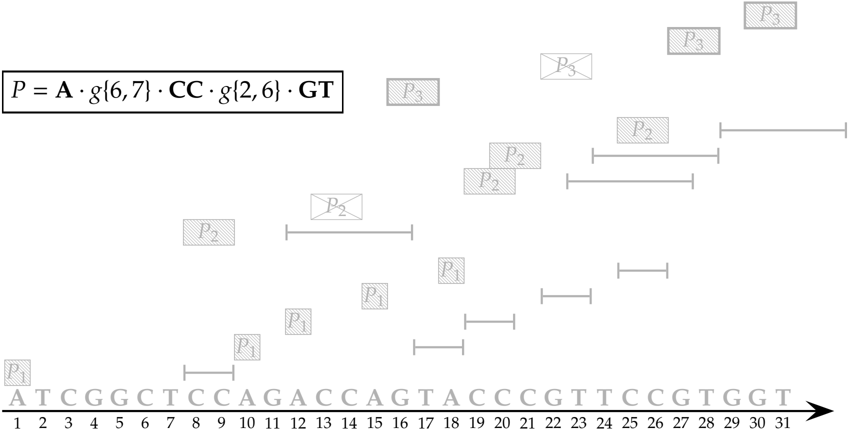
Can you get the best of both?

Illustrating the Algorithm

$$\boxed{P = \mathbf{A} \cdot g\{6,7\} \cdot \mathbf{CC} \cdot g\{2,6\} \cdot \mathbf{GT}}$$



L_2

L_3

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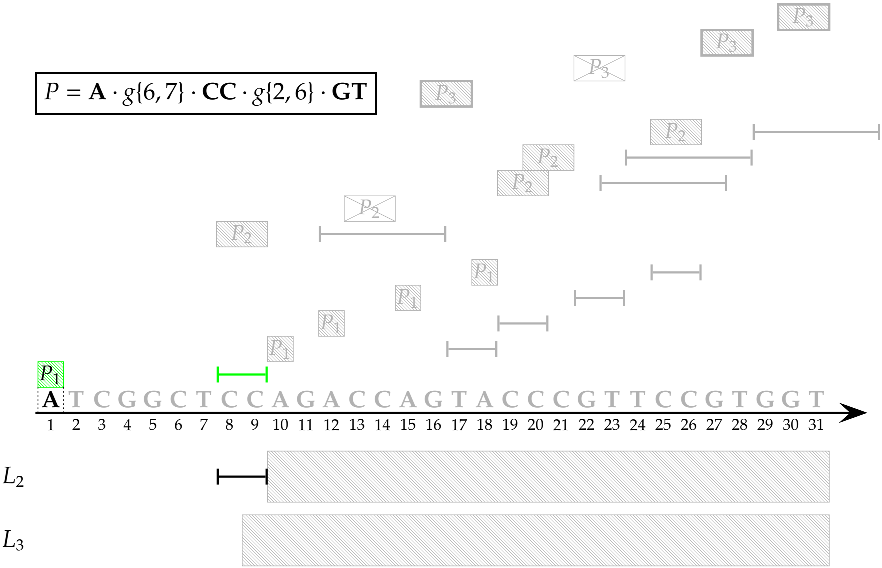

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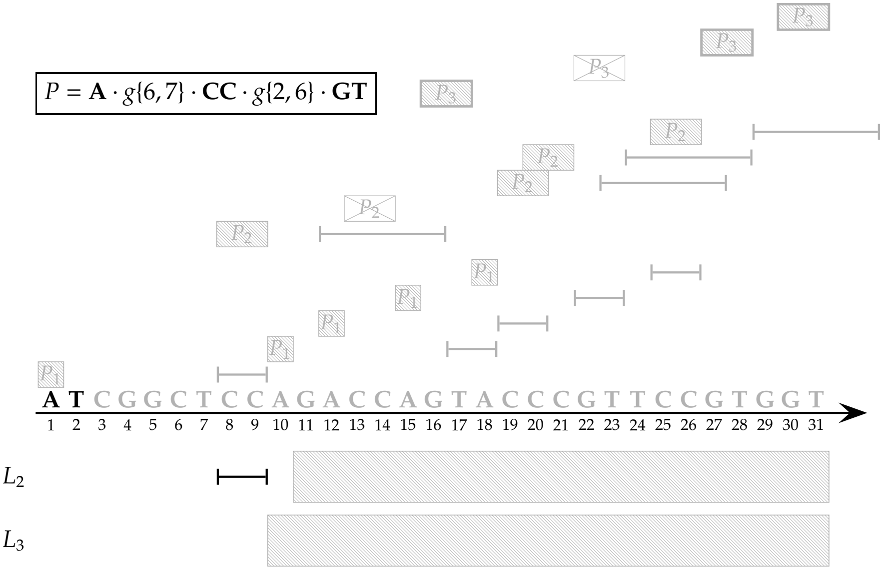

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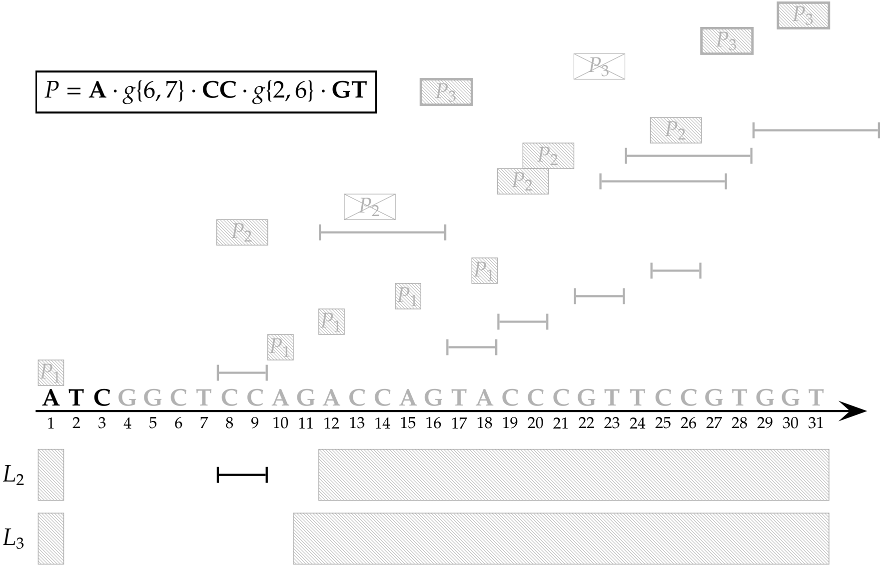

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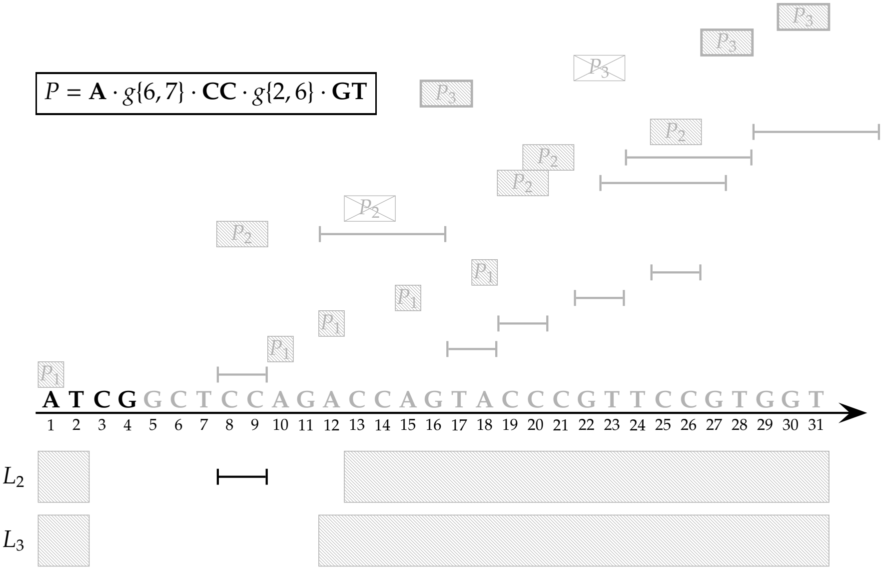

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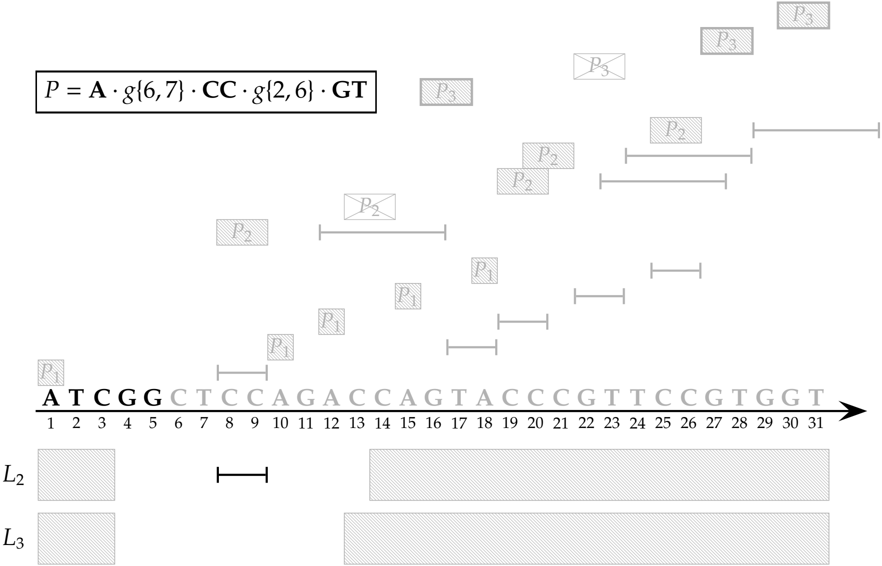

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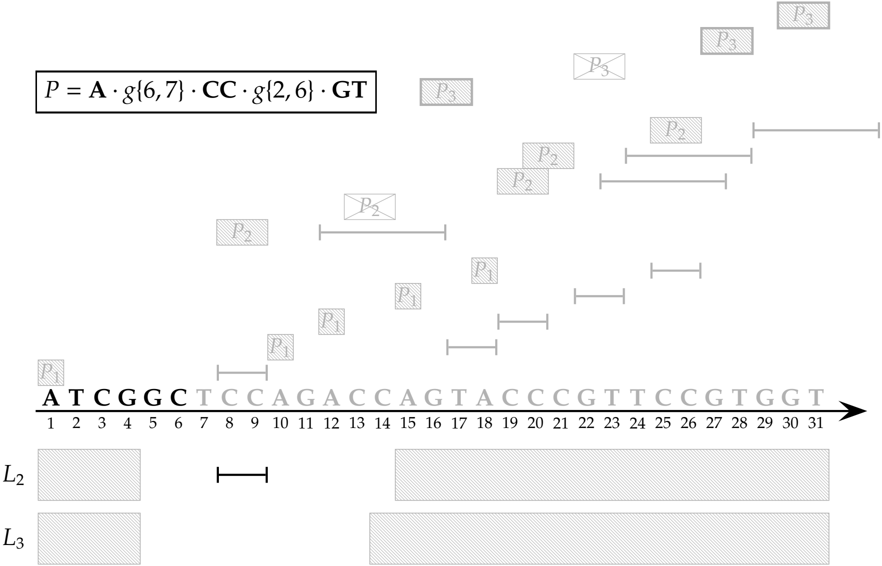

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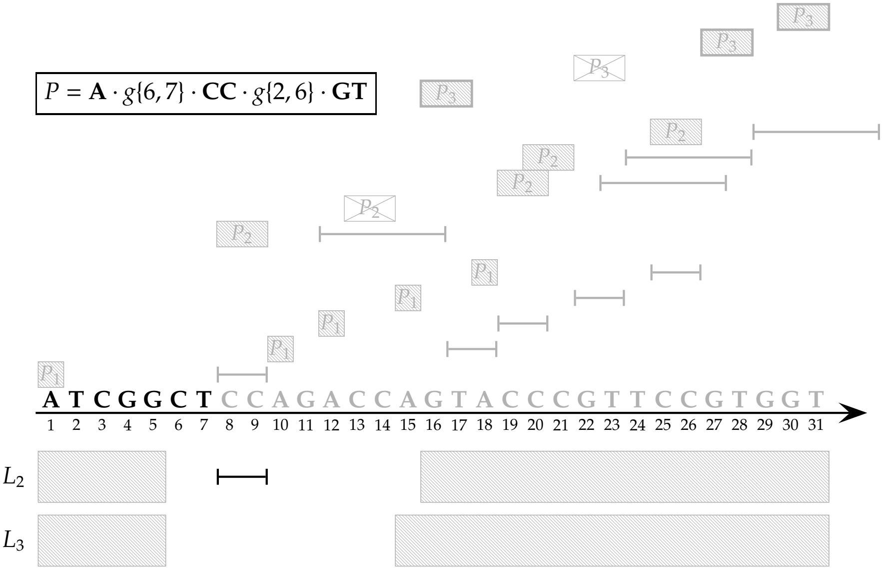

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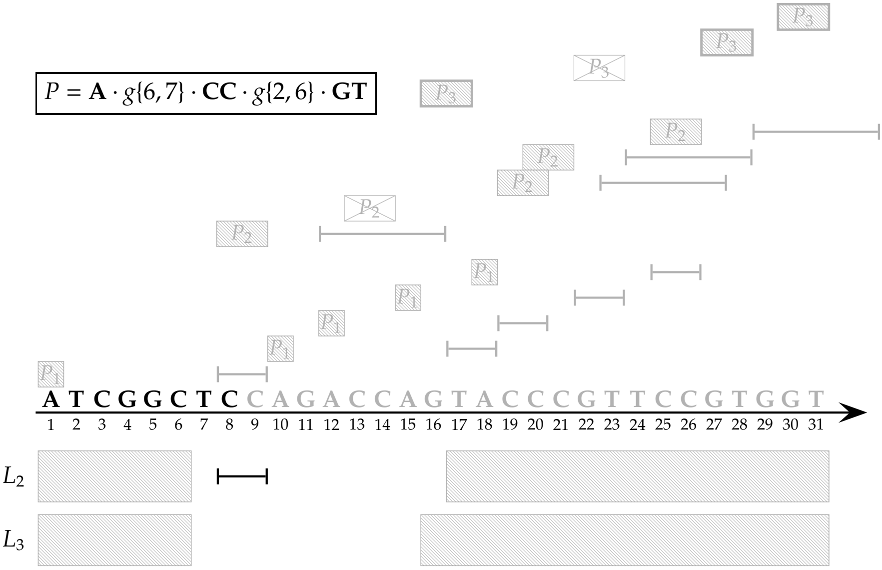

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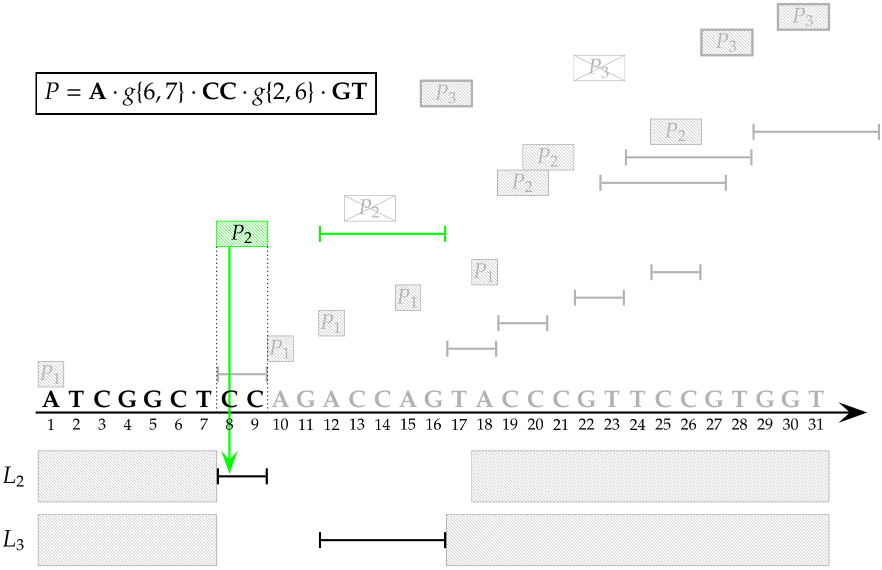

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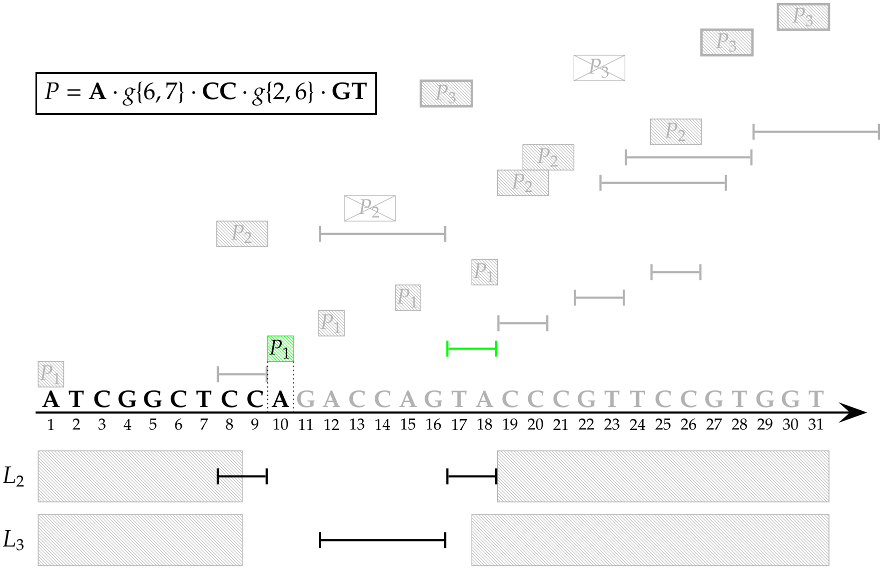

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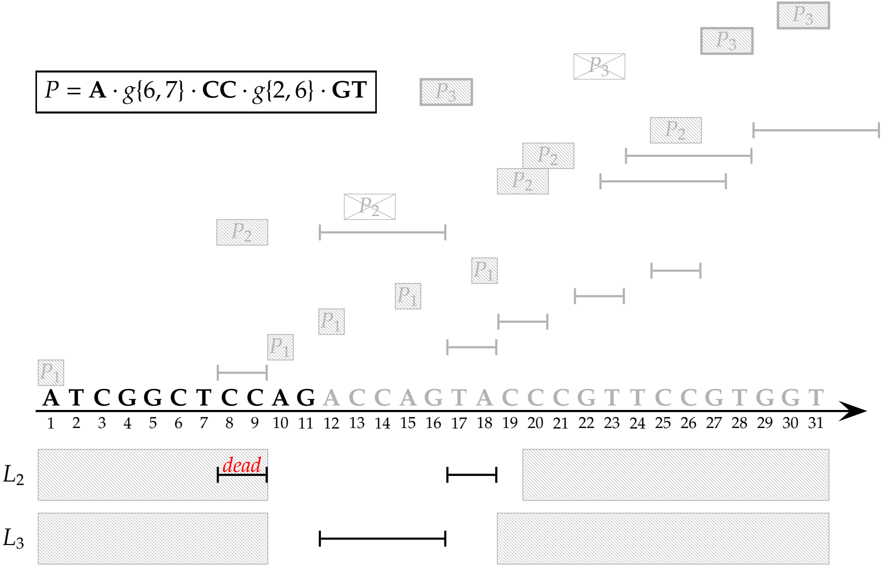

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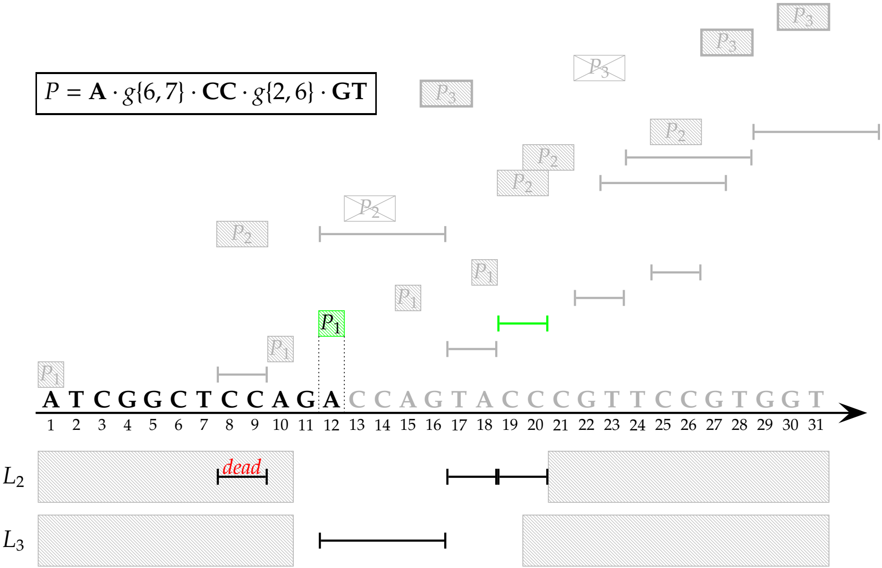

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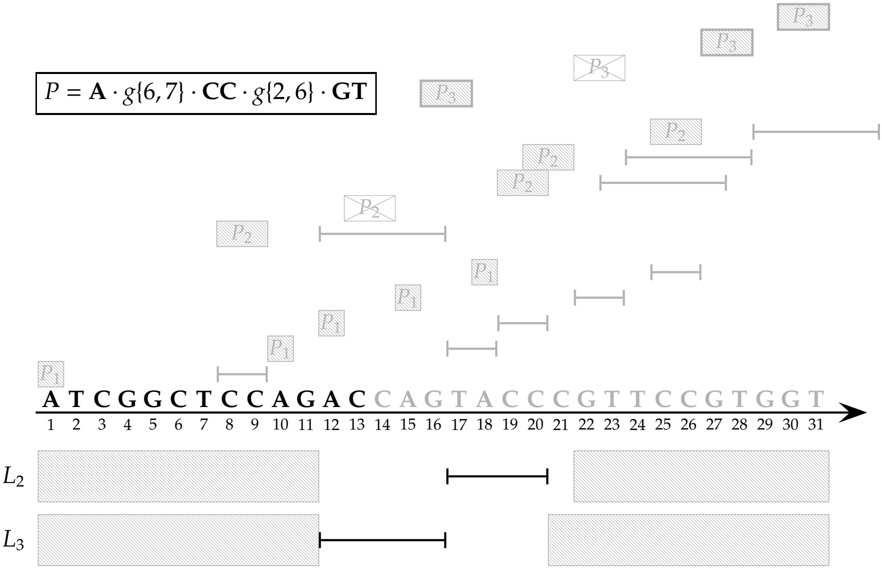

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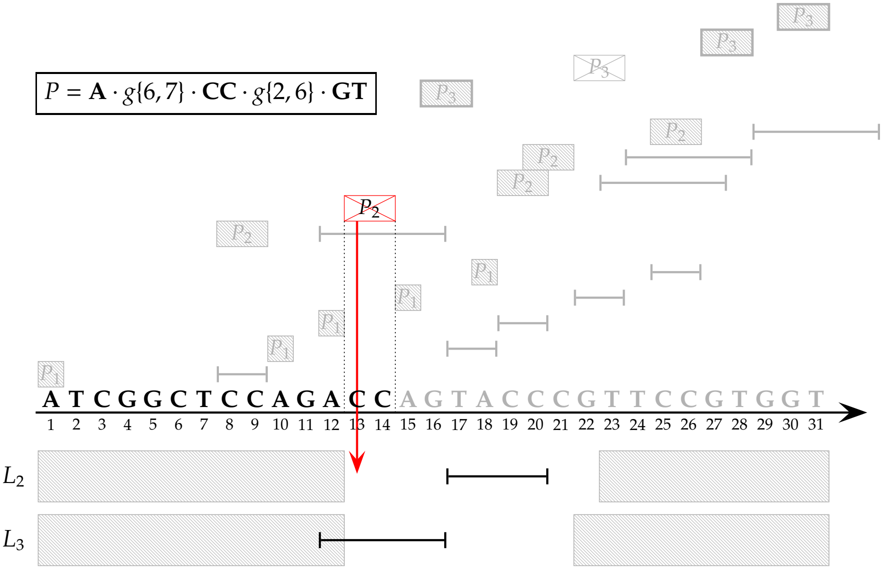

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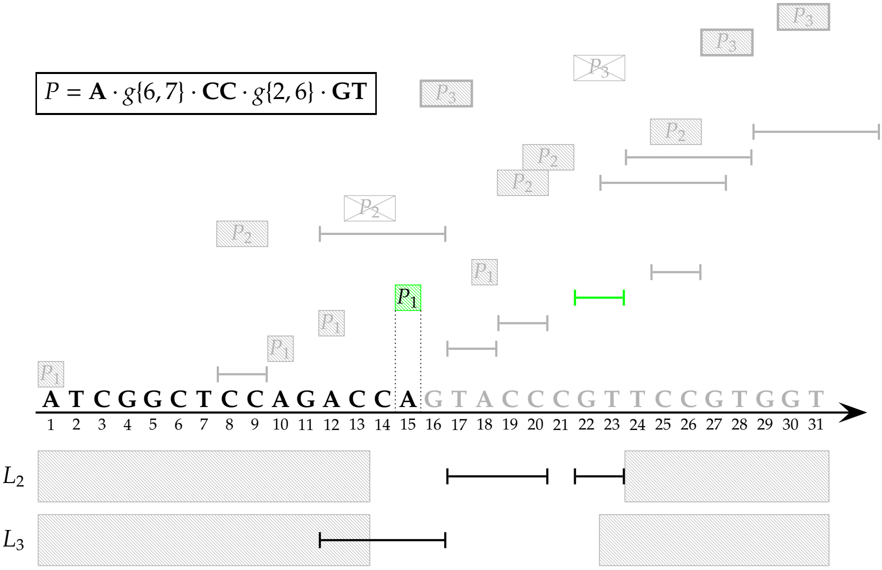

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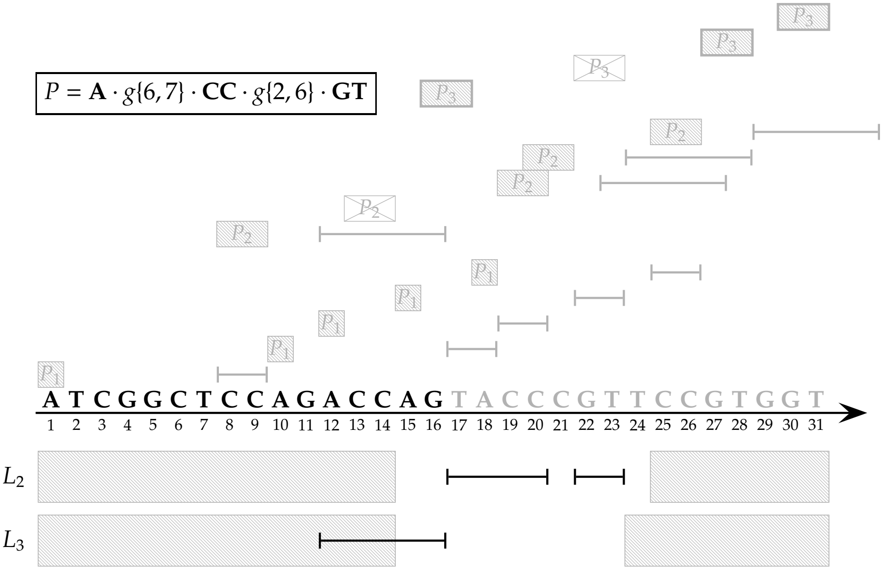

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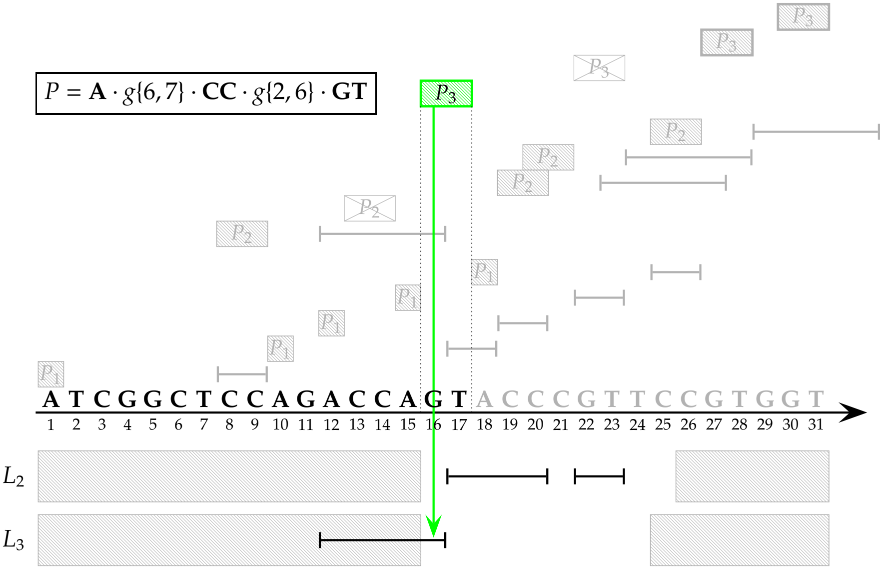

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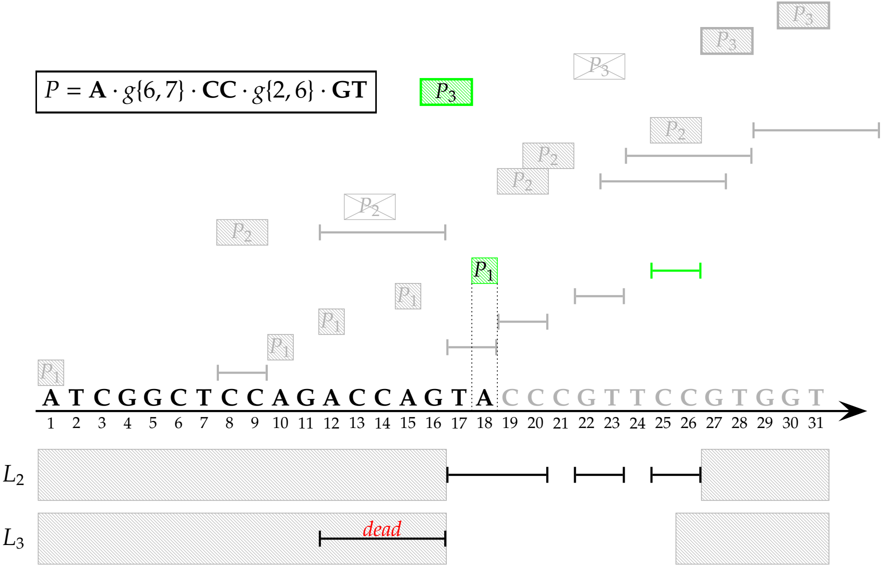

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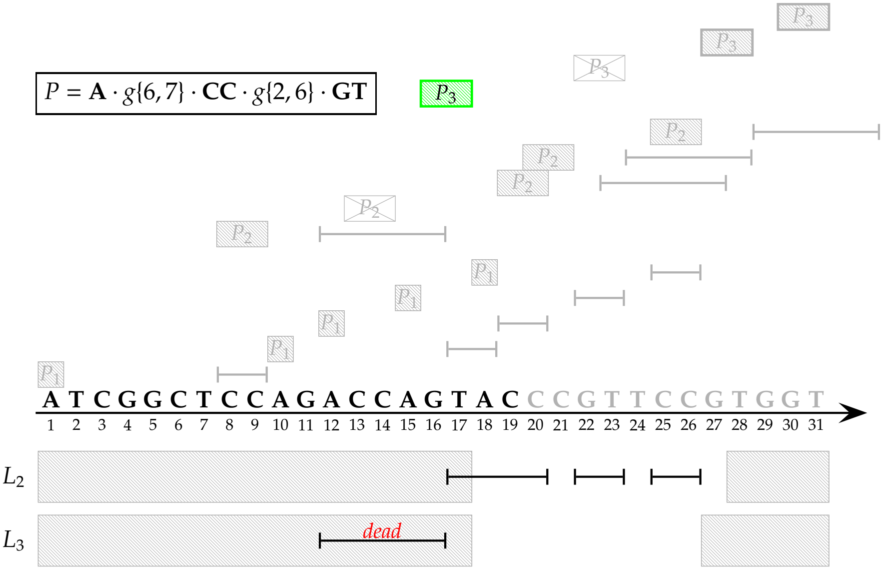

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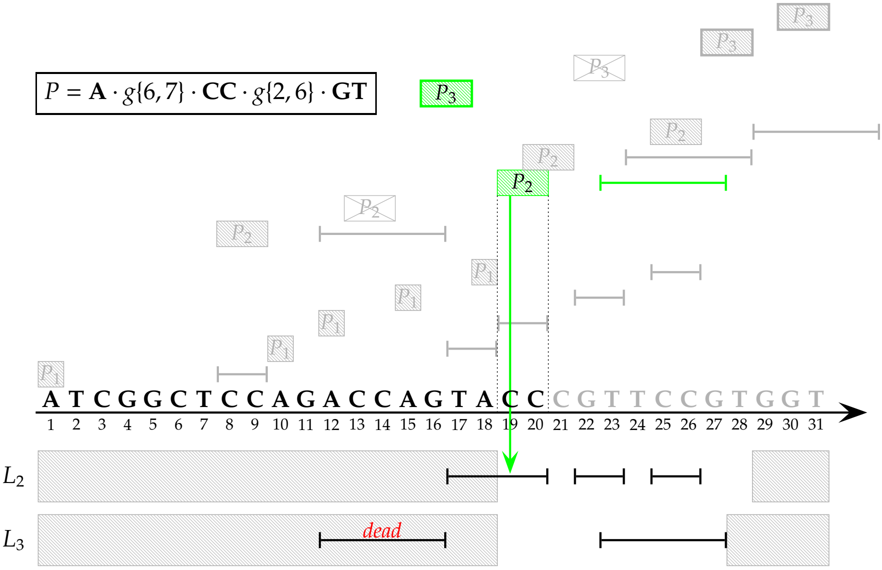

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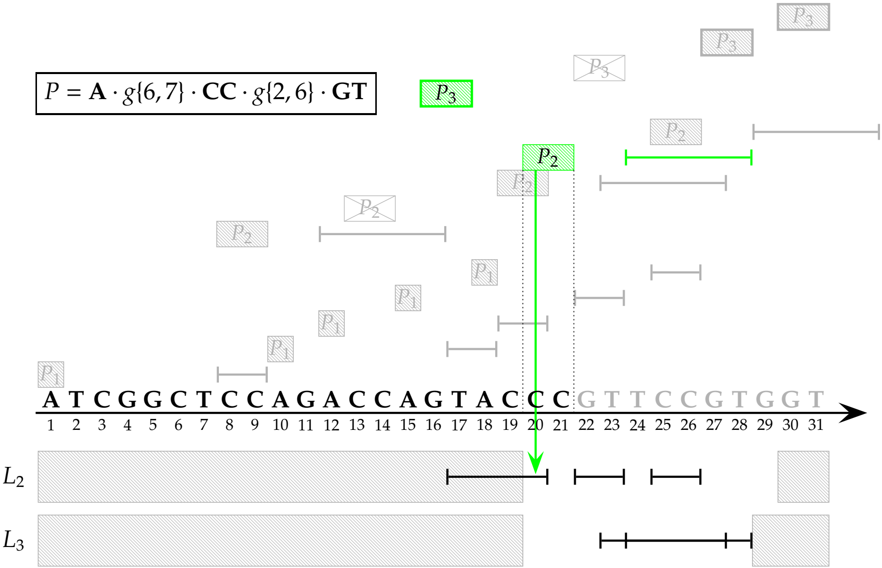

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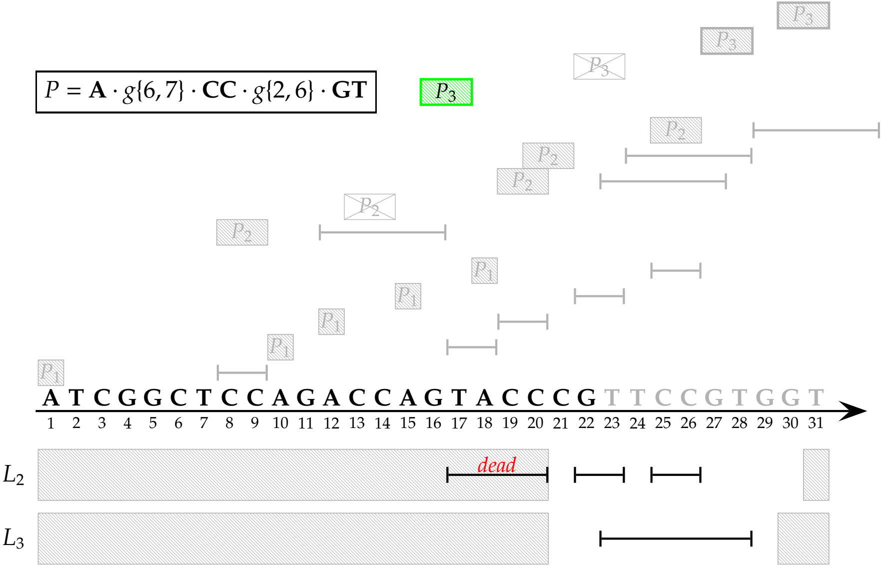

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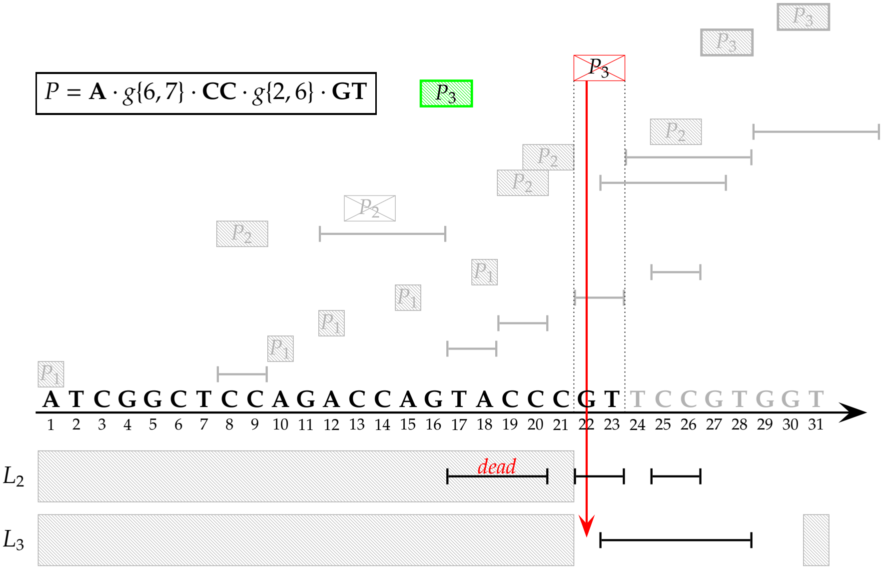

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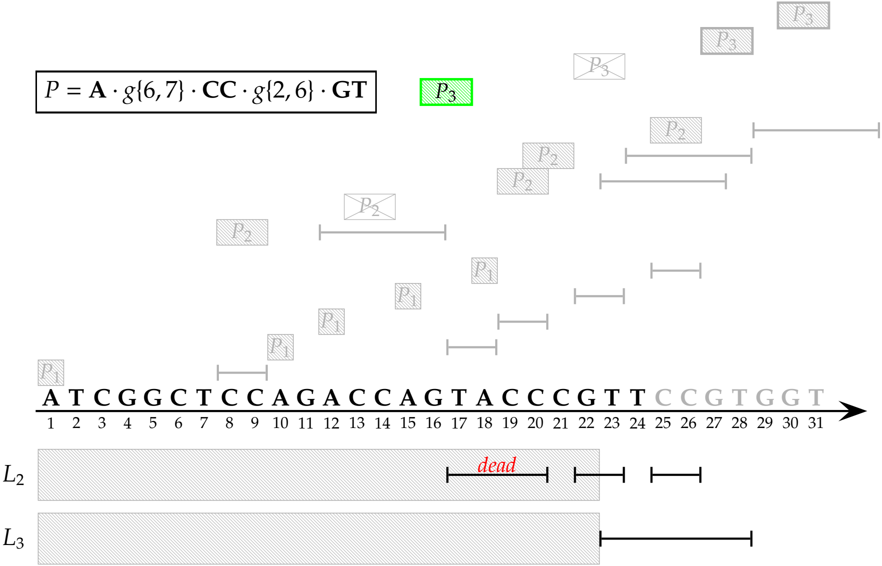

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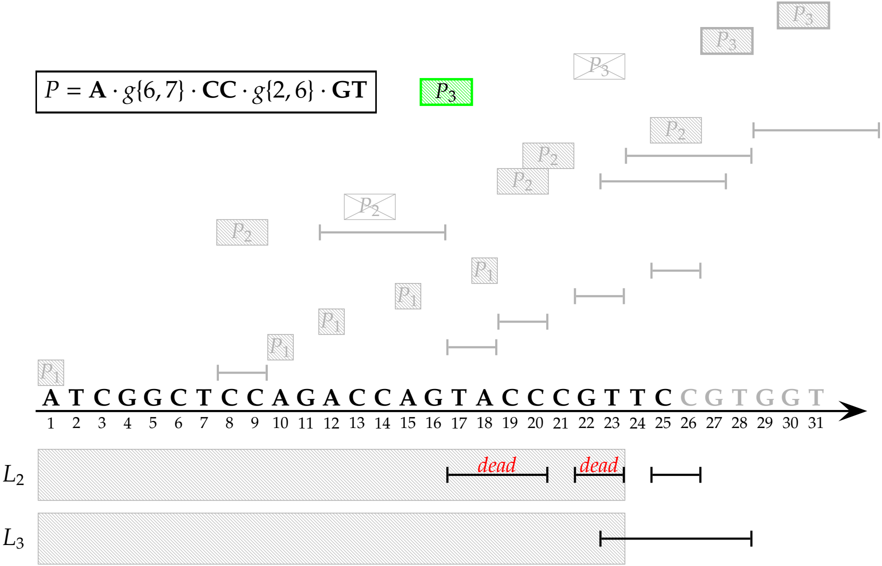

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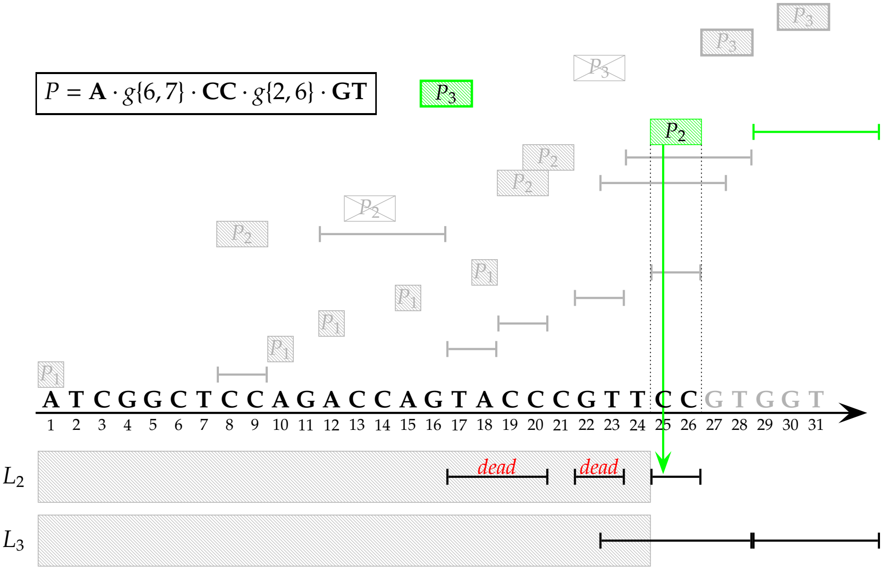

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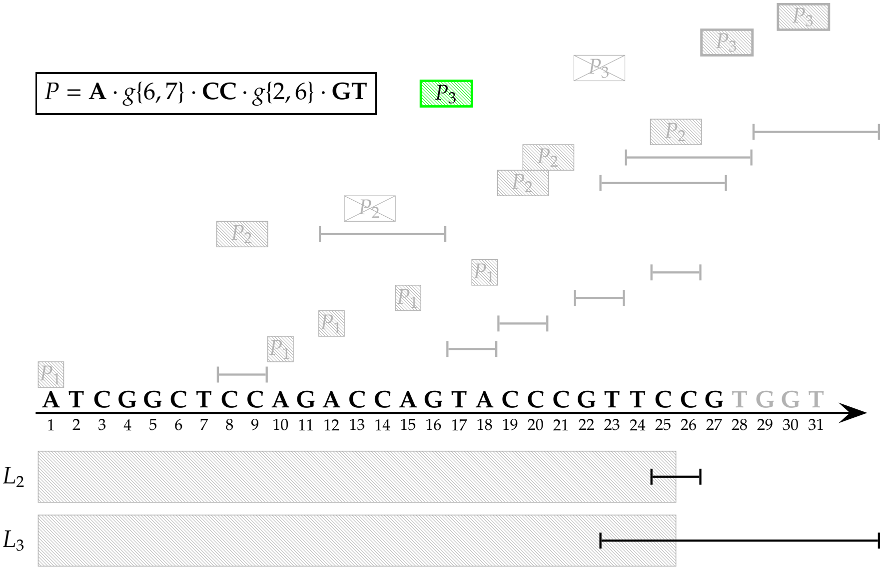

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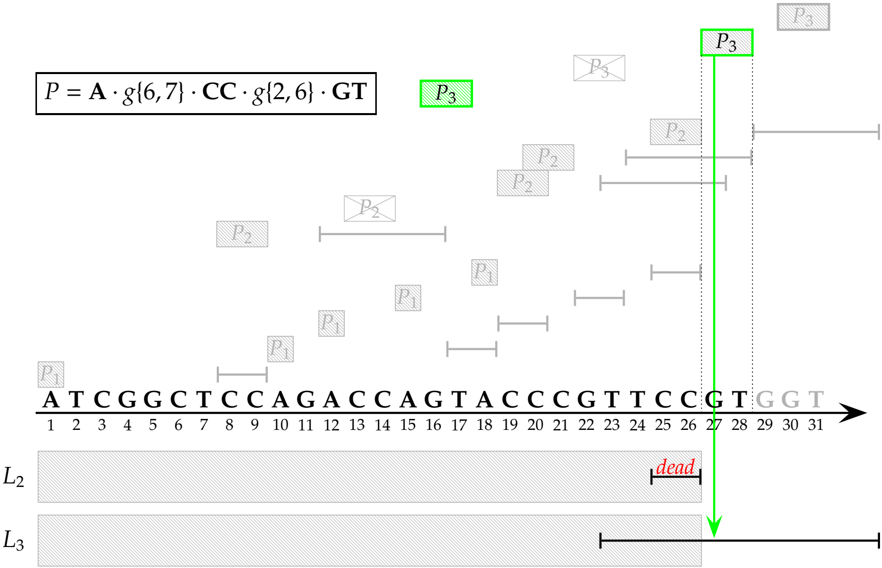

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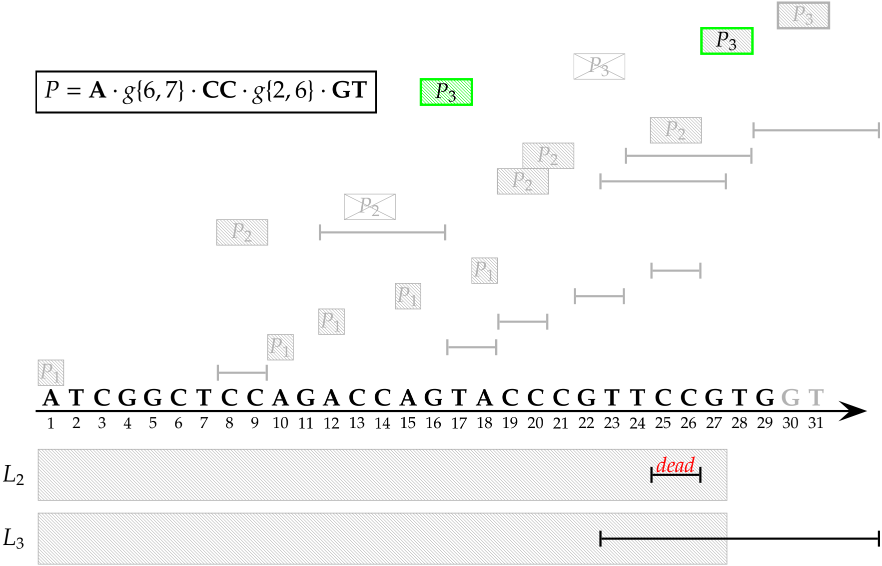

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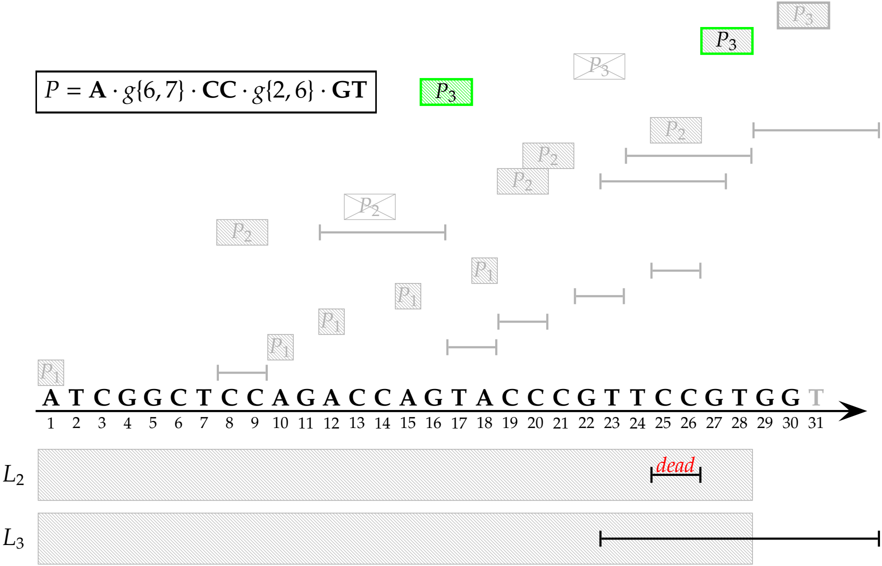

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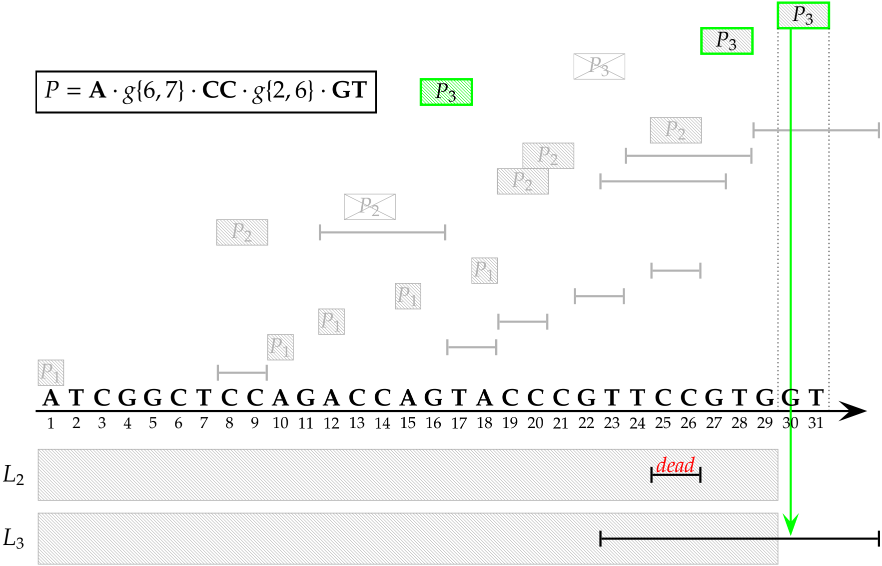

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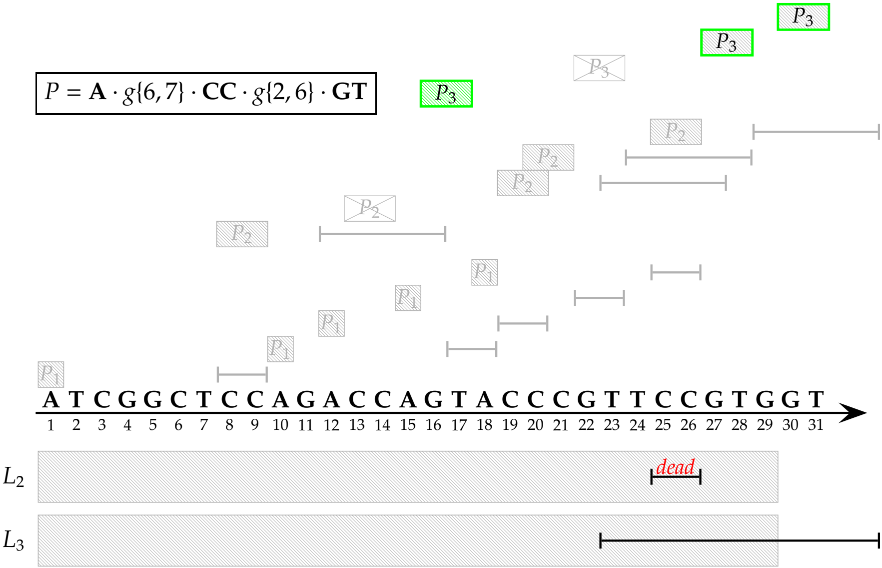

Time and Space

Claim: The algorithm runs in $O((n + m)\log k + \alpha)$ time and uses $O(m + A)$ space.

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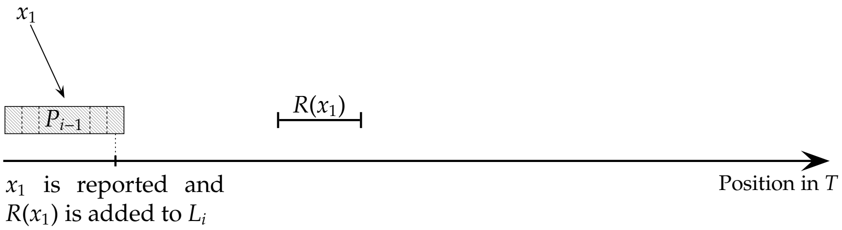
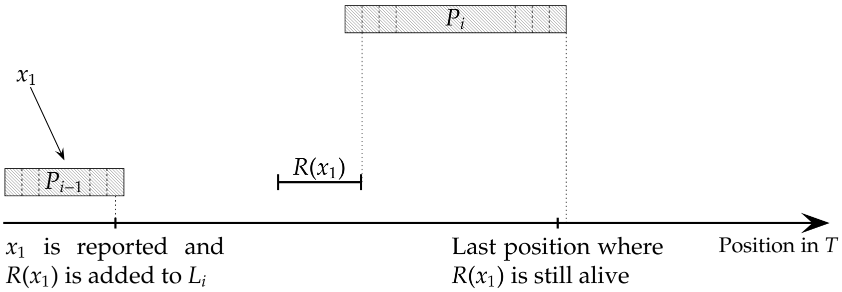
Space

- AC automaton takes $O(m)$ space.
- How much space is used by $L_2, \ldots, L_k$?

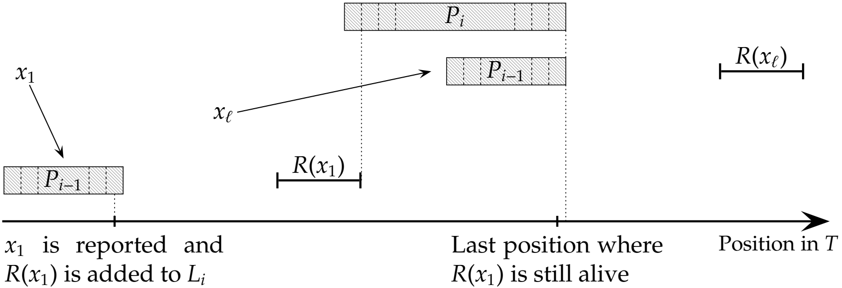
Maximum Size of L_i



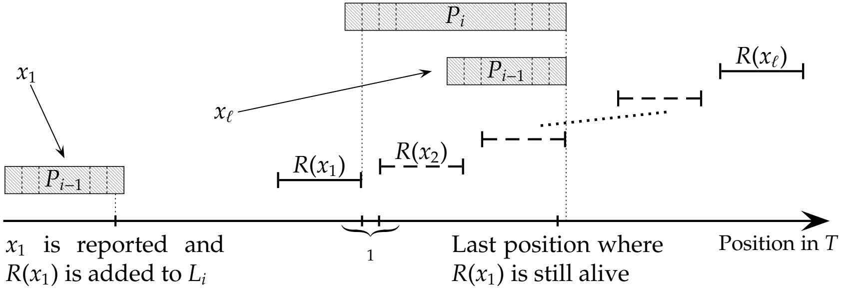
Maximum Size of L_i



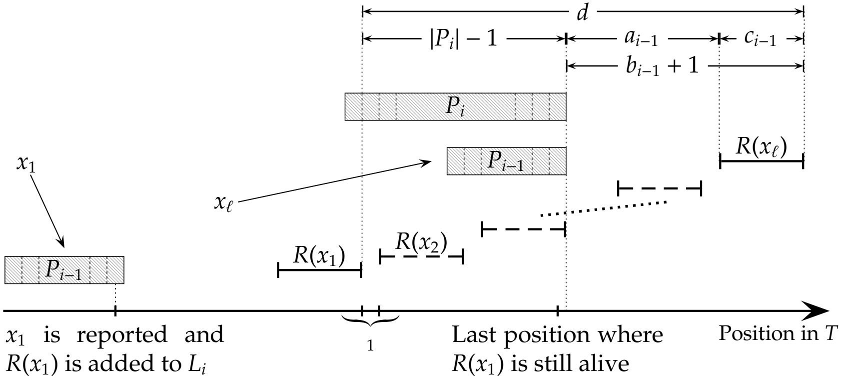
Maximum Size of L_i



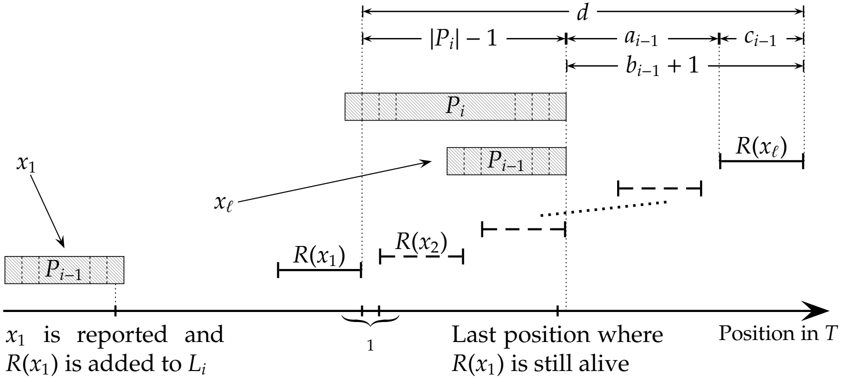
Maximum Size of L_i



Maximum Size of L_i

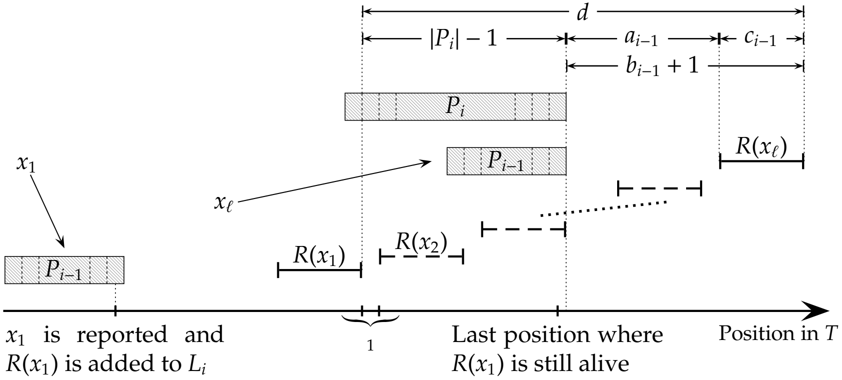


Maximum Size of L_i



$$|L_i| \leq \left\lfloor \frac{d}{c_{i-1}+1} \right\rfloor + 1 = \left\lfloor \frac{2c_{i-1} + |P_i| + a_{i-1}}{c_{i-1}+1} \right\rfloor = O(|P_i| + a_{i-1}) .$$

Maximum Size of L_i



$$|L_i| \le \left\lfloor \frac{d}{c_{i-1} + 1} \right\rfloor + 1 = \left\lfloor \frac{2c_{i-1} + |P_i| + a_{i-1}}{c_{i-1} + 1} \right\rfloor = O(|P_i| + a_{i-1}) .$$

Total space: $\displaystyle \sum_{i=2}^{k} |L_i| = O\left(\sum_{i=2}^{k} |P_i| + \sum_{i=1}^{k-1} a_i \right) = O(m + A)$